Effect of Thickness Stress in Stretch-Bending

A.H. van den Boogaard\textsuperscript{a}, W.C. Emmens\textsuperscript{b}\textsuperscript{*} and J. Huétink\textsuperscript{a}

\textsuperscript{a}: Univ. of Twente, P.O. Box 217, 7500 AE Enschede, the Netherlands
\textsuperscript{b}: CORUS R\&D, P.O.Box 10 000, 1970 CA IJmuiden, the Netherlands
*corresponding author, w.c.emmens@utwente.nl

Abstract. In any situation where a strip is pulled over a curved tool, locally a contact stress acts on the strip in thickness direction. This contact stress changes the stress state in the material, which will influence the deformation. One effect is that the yield stress in the plane of the strip is reduced. Predictions by a simple model agree with observation from a 90-degree bending test found in literature, and indirectly with observation from a stretch-bend test found in literature. The change in stress state also affects the formability. This is analyzed by applying the maximum force condition on this situation. The predictions agree with a more thorough analysis of the effect of thickness stress in general, but the predictions of both methods are lower than actually observed in tests.

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INTRODUCTION

The common FLC is valid only within certain limitations, one of them is the absence of bending and normal stresses. The occurrence of bending in a forming operation can cause several effects, the most relevant of which is the raising of the formability. The latter is for example encountered in the measurement of FLCs using Nakazima strips on tools of various radii.

If a strip is pulled over a curved tool there is a contact stress acting on the strip as pictured in Fig. 1. Tangential tractions are usually avoided as much as possible e.g. by lubrication, and will be ignored here. The contact stress changes the stress state in the material, so that it cannot be considered as a plane-stress state anymore. For the limited curvatures used in most types of sheet forming that effect seems to be ignored in the literature. A well known effect is that a change in stress state affects the yield stress in tension in the plane of the strip. A reduced in-plane stress also affects the stability of deformation (necking) and hence, the formability. Both effects will be discussed here, and will be compared to results found in literature.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Bending over a radius.}
\end{figure}
EFFECT OF A NORMAL PRESSURE ON THE IN-PLANE TENSION

For the situation in Fig. 1 it can be shown from equilibrium equations that $\sigma_{3,i} = -\sigma_{1,\text{mean}}/R$ where $\sigma_{3,i}$ is the normal contact stress at the inner side of the bend strip ($\sigma$ is compressive, so negative). The magnitude of $\sigma_3$ will vary over the thickness of the strip, but we will simplify matters and only refer to the mean value. In general we can write $\sigma_{3,\text{mean}} = \sigma_{3,i}/a$ where however the constant $a$ may depend on the material and the geometry, so:

$$\sigma_3 = -\sigma_1 \frac{l}{aR}$$

(1)

If $\sigma_3$ would vary linearly we would get $a = 2$ but this is not known beforehand. In this paper several values for $a$ will be used, but note that these are arbitrary. The effect of change of stress state is that it affects the yield stress in tension $\sigma_1$. Tresca's criterion simply states $\sigma_1 - \sigma_3 = \sigma_f$ where $\sigma_f$ is the material's flow stress. This yields:

$$\sigma_1 = \sigma_f / (1 + t / aR)$$

(2)

This shows that the yield stress in tension will reduce if there is bending over some radius, because of the introduction of a normal stress.

90-Degree Bend Test

In literature several types of bending-under-tension tests are reported. One is a bending over a 90° radius under back-tension as pictured in Fig. 2, left. This type of test simulates the pulling of material over a die radius, and has been used for a variety of reasons, for example in tribology research. Several researchers have reported detailed results for this kind of test, but generally report pulling forces and not stresses. If forces are measured at the same elongation (read: reduction of thickness) they may be interpreted as stresses but in general this is not known. So the results have to be interpreted with care.

Wagoner and co-workers have used this type of test recently on DP steel [1,2] and have indeed observed a reduction in pulling force, and have published the normalized maximum pulling stress (= measured stress / UTS) as a function of $R/t$; their results are compiled in Fig. 2, right. The general relation predicted by (2) agrees with the observed data, notably with the simulation for series 2. Note also that formula (2) even with $a = 3.5$ seems to over-estimate the effect for some series.

FIGURE 2. Left: principle of 90-degree bend test with back-tension, $F_2>F_1$. Right: comparison to literature data. 1 = measured data from [1], 2,3: data from [2], both measured (points and dashed lines), and simulated (solid lined). Thick black line: relation according to formula (1) with $a = 3.5$. 
Hudgins has carried out similar experiments on DP steel subjected to different heat-treatments [3]. He has published values of the maximum pulling stress, that have been converted here to normalized stress by dividing the value by the material’s UTS also published by Hudgins, the results are shown in Fig. 3. There is a good agreement between the measured data and the relation according to (2), with the exception of series 6. Series 6 refers to a material that after heat treatment showed very little ductility, meaning that the UTS is measured at a very low elongation. So it is quite possible that the maximum force in the bending tests occurs at a different elongation than the measured UTS, but this is not known. It is noteworthy that Hudgins models the relation by fitting two straight lines, and concludes that the values for series 1 and 2 at $R/t \approx 4$ are ‘outliers’. However these data agree very well with relation (2).

In the above we have ignored thinning of the strip as a function of its elongation and have taken $t$ in (2) as constant. However this is a second order effect that does not affect the conclusions in this section. Correctly calculated forces will be presented in Fig. 8 below in one of the next sections.

**Stretch-Bend Test**

A second type of test with bending and contact stresses is a stretch-bend test where a clamped strip is bent by pushing a cylindrical tool at its centre as shown schematically in Fig. 4. This is the simplest test as any movement of the strip over the radius can be ignored.

Experiments have shown that there is a difference between the strain in the straight part and in the bend part and this will be analyzed here. Part A is not supported; part B however is pulled against the punch, and therefore its strain state is affected. At the interface $X$ there must be an equilibrium of forces, so: $F_A = F_B$. As part B requires a lower tensile force for stretch, it must be stretched to a higher level than part A to obtain equilibrium. Common to what is used in the stability criterion for local necking, we will apply this equilibrium on a piece of unit width, so $F = \sigma t$ and the condition becomes: $\sigma_A t_A = \sigma_B t_B$.

Assuming a power-law hardening relation we can write:

$$\sigma_A = C \varepsilon_A^n, \quad \sigma_B = C \varepsilon_B^n \frac{1}{1 + \varepsilon_B^0 / aR}, \quad t = t_0 \varepsilon^3$$

For any given strain in part A ($\varepsilon_A$) the strain in part B can now be calculated, depending on the tool radius $R$, the original sheet thickness $t_0$, the hardening coefficient $n$, and the strain state determining the relation between thickness strain $\varepsilon_t$ and length strain $\varepsilon_l$. 

**FIGURE 3.** Comparison to literature data. All series (1-6) are measured data from [3], some series are incomplete. Thick black line: relation according to formula (1) with $a = 3.5$. 

![Graph showing normalized pulling stress vs. $R/t$](image)
The strain distribution in some actual tests has recently been determined and published by Kitting et al. [4]. The results obtained with a low bending radius showed a high amount of strain localization which makes them unsuitable for our analysis. So we will analyze only the results obtained with a high bending radius (R20), and only in cases that showed a fairly uniform strain in part B. The strain state is approximately uniaxial, therefore we will use $\varepsilon_3 = -\varepsilon_1/2$. Note that Kitting has measured the strain at the outer surface, so the strains predicted by our model in part B have to be corrected for the curvature of the strip. The results are presented in Fig. 5 using $t_0 = 1.5$ mm and $n = 0.14$, in which only the strain levels predicted by the model are to be considered, not the transitions. There is a good agreement between model and experiments, for low bending depths the model with $a = 5$ seems to underestimate the effect, but for larger bending depths the model seems to overestimate the effect.

**EFFECT OF A NORMAL PRESSURE ON FORMABILITY**

A contact stress also has an effect on the formability. Popular speaking: a normal pressure eases the elongation of the material, meaning that a tensile instability in deformation occurs later than without thickness stress.

**General Considerations**

A detailed and general analysis is not easy. A very thorough M-K analysis was recently carried out and published by Allwood and Shouler [5], and some of their findings are presented in Fig. 6. This figure shows clearly that the formability increases when a thickness stress is applied. For the plane-strain situation the effect is approximately linear and can be written as:
where $\varepsilon_0$ is the necking limit at plane strain conditions. This relation is confirmed by other models not mentioned here.

It is difficult to check these findings with results from bending tests. Kim has published values for 'displacement to failure' for his tests [2] but these cannot be simply converted to actual strains. Kitting has presented actual measured strains at fracture [5], and the strain state in tests with small bending radii are approximately plane-strain. However, the measured strains are considerably higher than predicted by formula (4), notably in cases of low bending radius. It must be kept in mind that models like the one developed by Allwood or presented in the next section predict the strain at onset of necking, while Kitting has presented actual measured values. This may be an effect of measurement or definition of limit strain. Also, for materials that show considerable strain-rate hardening the actually measured strains are expected to be higher than the strains at the onset of necking predicted by simple models.

Another possibility, however, is that there are other mechanisms that postpone the tensile instability, and one will be presented here. If there is indeed a severe neck the material in the neck may loose contact with the tool as shown in Fig. 7 at location A. This means that there is no thickness stress any more, and the 'softening' of the material at B does not take place at A. Consequently at A the material becomes stronger than at B, and this will slow down further development of the neck. However this mechanism is speculative, and it is not known if it does actually happen in a practical operation.

FIGURE 6. Effect of thickness stress on formability. Data from [5], $q = -\sigma_3/\sigma_1$,

$$\frac{\varepsilon_0(\sigma_3)}{\varepsilon_0(0)} = 1 + \frac{-\sigma_3}{\sigma_1}, \quad (4)$$

FIGURE 7. In case of a severe neck (at A) the material looses contact with the tool.
Analysis with the Maximum Force Condition

The effects of contact stress on formability can also be analyzed by applying the so-called maximum force condition. This will be done here, albeit slightly simplified.

For the onset of local necking the maximum force condition has to be applied on a piece of unit width, so, using (2):

\[ dF = 0 = d(\sigma_I t) = d(\sigma_I \frac{t}{1+\alpha t}) = d\sigma_I \frac{t}{1+\alpha t} + \sigma_I dt \frac{1}{(1+\alpha t)^2}, \quad \alpha = \frac{1}{aR} \]  

Hence:

\[ \frac{d\sigma_I}{\sigma_I} = -\frac{dt}{t} \frac{1}{1+\alpha t} = -\frac{de_3}{1+\alpha t} \]  

Now using \( \varepsilon_3 = -(1+\beta)\varepsilon_1 \) (\( \beta \) defined conventionally as \( \varepsilon_2 = \beta \varepsilon_1 \)):

\[ \frac{d\sigma_I}{\varepsilon_1} = \sigma_I (1+\beta) \frac{1+\alpha}{1+\alpha t} \]  

Note however that in this equation \( t \) is not constant, but \( t = t_0 e^{\varepsilon_3} \). Assuming now that the strain state does not vary too much, we can write as an approximation \( \varepsilon_1 \approx C e_1^n \), and by this the equation can be solved:

\[ C n \varepsilon_1^{n-1} = C e_1^{n+1} \frac{1+\beta}{1+\alpha t} \]

\[ n = \varepsilon_1 \frac{1+\beta}{1+\alpha t} = e_1 \frac{1+\beta}{1+\alpha t_0 e^{-(1+\beta)\varepsilon_1}} = -\frac{\varepsilon_3}{1+\alpha t_0 e^{\varepsilon_3}} \]

\[ \frac{-\varepsilon_3}{n} = 1+\alpha t_0 e^{\varepsilon_3} \]

This is a transcendental equation that has to be solved numerically. The outcome is \( \varepsilon_3 \) at the onset of local necking as a function of \( n \) and \( \alpha t_0 = t_0/aR \).

If \( \varepsilon_3 \) is not too large we can make a first approximation by applying \( e^{\varepsilon_3} \approx 1+\varepsilon_3 \):

\[ \frac{-\varepsilon_3}{n} = 1+\alpha t_0 (1+\varepsilon_3), \quad \varepsilon_3 = -n \frac{1+\alpha t_0}{1+n\alpha t_0} \]  

For a situation of plane strain we have \( \varepsilon_3 = -\varepsilon_1 \), and setting as a further approximation in (9) \( t \approx t_0 \), valid only for situations of \( \varepsilon_1 \ll 1 \), we get:

\[ \varepsilon_1 \approx n(1+\alpha t_0) = n(1+\frac{t_0}{aR}) \]  

This is in fact the same relation as deduced from the Allwood results expressed in Equation (4), note that \( t_0/aR = -\sigma_3/\sigma_1 \) (see Equation 1).

Equation (8) has been solved for some cases and the results are shown in Fig. 8. The parameter 'aR/40' can be interpreted as the ratio of mean thickness stress and mean pulling stress at zero elongation (the onset of the operation). In an actual stretch-bending operation the value of that parameter will not likely be lower than 10.
The upper part shows the increase of formability, in fact the effect on the thickness strain at the onset of necking; this does not depend on the strain state. The simple model is just Equation (10) that can be interpreted as a limiting situation for \( n \to 0 \), the other two curves have been derived from Equation (8). There is some influence of \( n \): the effect reduces for increasing \( n \), this can also be seen in the simplified Equation (9). The effect is limited in size, for \( aR/t_0 = 10 \) it is less than 10%.

In addition, the lower part of the figure shows the effect on the UTS, or better: the pulling force at \( \varepsilon_1 = n \), for a situation of constant thickness, and correctly for two strain states (uniaxial: for isotropic material). The fact that the uniform strain actually increases by bending is neglected here. The difference between the three situations is caused by the amount of thinning of the strip which is the highest for plane strain. The effect of \( n \) is even lower than was found for the necking limit.

Note that the effects are roughly each other's inverse, so that as a rule of thumb: if the force decreases by \( P \% \), then the necking limit increases by \( P \% \).

**DISCUSSION AND CONCLUSION**

In the previous sections a simple model has been developed that predicts certain phenomena occurring while bending a strip by looking at the change in stress state due to the contact pressure. The model showed good agreement with observations found in literature. However the model was developed with some simplifying assumptions:

- the phenomena assumingly depend only on the mean thickness stress \( \sigma_3 \) (equation 1);
- Tresca's yield criterion is used ignoring any effects of the transverse stress \( \sigma_2 \) (equation 2);
- values of 3.5 and 5 for the parameter \( a \) are used without any motivation, other than that the results look good.

Furthermore, some second order effects have been ignored. Consequently, one cannot simply conclude that the model is correct. However, the intention of this paper is not to present a reliable model, but only to study possible effects that can be caused by the change in stress state created by the tool contact. The conclusion is that even modest values of the thickness stress, in this case as indicated by values for \( a \) of 3.5 and 5, can cause effects very much similar to effects observed in actual experiments, at least concerning the stresses and forces.

Therefore the overall conclusion is that the change in stress state in a situation of bending over a tool radius caused by a normal stress at the contact should not be ignored. Nevertheless, more research is still required to fully understand the increased formability in a situation of combined bending and stretching.
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REFERENCES