

Aiming for the Bull's Eye: Inflation Targeting Under Uncertainty*

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Abstract

We study the implications of uncertainty on the Central Bank's ability to achieve its objectives. Assuming multiplicative uncertainty in a standard forward looking model, we show that while the role for policy is reduced in line with Brainard's attenuation effect, at the same time, the contribution of expectations to the final outcome increases. This implies that unless expectations are exactly on target, the presence of uncertainty increases the gap between actual inflation and the inflation target. If the Central Bank is not always aware of how expectations are formed, this effect can harm its credibility and more generally its ability to steer the system following a shock. With this in mind, we thus look for an algorithm that removes this effect. To this end, we put forward a two-step algorithm in which the inflation target is state contingent. The Central Bank sets (as an auxiliary step) a variable inflation target that depends on both the degree of uncertainty, as well as the shocks that occur. We show that such an algorithm increases the level of variability in the system but for small levels of uncertainty the cumulative benefits of pinning down expectations more than compensate the costs of having to overuse the instrument. We demonstrate this through Monte Carlo Simulations.

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1 Introduction

The benefits of inflation targeting in the Svensson (1999) sense amount to providing a nominal anchor that acts as a focal point for private sector expectations. For the Central Bank (CB), on the other hand, inflation targeting provides an implicit commitment mechanism which increases its cost of deviating from announced targets and hence discourages it from doing so. The economy on the whole benefits from greater transparency because it leads to greater credibility and by consequence to effective monetary policies. From a political economy standpoint therefore, the literature associates the concept of inflation targeting with greater transparency and hence with more credible and effective policies. By the same token, a central bank that fails to achieve the target that it sets (and announces) will be penalized with a loss in credibility and hence a subsequent reduction in the ability to pursue its objectives. As Bernanke and Mishkin (1997) indicate, "... announcements alone are not enough; the only way to gain credibility is to earn it".

Our paper is motivated from the importance that inflation targeting puts on achieving the monetary objective announced. We analyze this issue in the context of an economy that is characterized by transmission uncertainty as modelled by Brainard, (1967). In describing the attenuation effect he puts forward, we observe that as the contribution of policy to the final inflation outcome reduces in the presence of multiplicative uncertainty, that of expectations increases proportionally to the prevailing degree of the specified uncertainty. Naturally, if the Central Bank operates under commitment, expectations are anchored by the level of inflation the Central Bank aims at. In a discretionary environment where expectations are parametric to the CB's actions, it is then the assumption of rational expectations (RE) that "forces" the private sector to adjust their expectations such that the outcome is consistent with the intentions of the Central Bank. The discrepancy in the way policy and expectations affect the final outcome is therefore redundant. However, if one departs from a rigid application of the RE paradigm, then multiplicative uncertainty can seriously compromise the CB's ability to attain its objectives. We will thus assume that there are instances for which the Central Bank is not aware of the exact formulation of expectations and designs policy based on accessing these expectations wrongly. Our example will thus include a policy derived under the assumption of rational expectations, when actually agents turn out to be backward looking. The presence of uncertainty will then add to the gravity of the issue as it gives a greater weight to expectations. The implication following this discrepancy is that the Central Bank is unable to close the gap between current inflation and its target, a gap that is only enhanced as uncertainty increases. We analyze two issues: *first*, if there is some value in attaining the target as inflation targeting proponents advocate, then we aim to find an algorithm that will both manage to achieve it on average, as well as still operate in an optimization framework,

so that the procedure remains transparent to the public. We thus identify a two-step algorithm according to which, in the first step, the central bank deviates from the target in order to reactivate the instrument and only in the second step, does it aim for the actual target itself. The two-step procedure amounts therefore, to the Central bank aiming *for* the bull’s eye and not directly *at* it. *Second*, we identify the conditions of uncertainty under which such an algorithm can prove superior in welfare terms to the Brainard result. This is important in an inflation targeting framework as announcing a target that is unlikely to be achieved is not necessarily increasing one’s credibility (Posen, 2002).

The paper is organized as follows. Section 2 discusses the model under certainty and multiplicative uncertainty respectively and shows how the impact of expectations on the final inflation outcome increases with uncertainty. Section 3 derives a two-step algorithm to inflation targeting that neutralizes this effect. Section 4 discusses under which conditions this algorithm can be desirable and shows this both analytically as well as with the aid of numerical simulations. Section 5 concludes.

2 Expectations and Uncertainty

Most of the attempts to examine the effects of uncertainty in a dynamic framework rely on a backward-looking set-up (Söderström, 2002, Srouf, 1999). The somehow surprising result, from the point of view of inflation targeting proponents, is that uncertainty in the structure of the economy implies that achieving the target is not optimal as it may lead to instability in the system. This seems at odds with the general perception that the main advantage of inflation targeting is that it stabilizes expectations (see the empirical evidence in Johnson, 2002 and Levin et al, 2003). The contradiction is only apparent: controlling expectations is relevant only if private sector expectations are an important determinant of the economic outcome. Thus the right set-up to test the effect of model uncertainty on the Central Bank’s ability to achieve its inflation objective is a forward-looking model. We apply therefore, a standard New Keynesian model as described in Clarida Gali and Getler (1999) and Woodford (2004) and used in a similar context by Giannoni (2002) in which expectations play again a central role (Woodford, 2003). Following Clarida, Gali and Getler (1999), the economy is thus described by the following pair of log-linear relations in which the variables are expressed in deviations from their steady state:

$$\pi_t = \beta E_t \pi_{t+1} + \alpha y_t + \varepsilon_t \tag{1}$$

$$y_t = E_t y_{t+1} - \gamma (\dot{i}_t - E_t \pi_{t+1}) + \xi_t \tag{2}$$

where (1) is an expectations-augmented “AS” relation in which present inflation is a function of private sector inflation expectations one period ahead, and (2) is an intertemporal “IS” relation. Notation follows convention and coefficients

satisfy, $\alpha, \gamma > 0$. Supply shocks are uncorrelated autoregressive processes, i.e.:

$$\varepsilon_{t+1} = \rho\varepsilon_t + v_{t+1}, \quad 0 < \rho < 1$$

and v_{t+1} has a zero mean and constant variance. We solve under the AS constraint in which output gap is considered the intermediate instrument. We thus abstract, for simplicity, from the issue of actual monetary policy transmission. The Central Bank minimizes the following objective function:

$$\min_y L_t = \frac{1}{2} E_t \sum_{\tau=0}^{\infty} \beta^\tau \left\{ (\pi_{t+\tau} - \pi^*)^2 + y_{t+\tau}^2 \right\} \quad (3)$$

The central bank announces that it aims at a level of inflation equal to π^* , known as the inflation target. This objective function (3) is transformed when knowledge of the structural equations (1) and (2) is not known. Assume now that there is limited knowledge about the monetary transmission mechanism. Similar to Brainard's contribution, this is represented by coefficient α in the AS equation being stochastic i.e.: $\alpha_t \rightarrow N(\bar{\alpha}, \sigma_\alpha^2)$ ¹ and has come to be known as Brainard Uncertainty. The existence of such uncertainty implies that the objective function (3) can now be expressed in terms of the first and second moments of the uncertain terms (see Appendix B for a detailed derivation):

$$\min_y L = \frac{1}{2} E \sum_{\tau=0}^{\infty} \beta^\tau \left\{ (\bar{\pi}_t - \pi^*)^2 + y_t^2 (1 + \sigma_\alpha^2) \right\} \quad (4)$$

where $\bar{\pi}_t = \beta E_t \pi_{t+1} + \bar{\alpha} y_t + \varepsilon_t$. The term $y_t^2 \sigma_\alpha^2$ now represents the extra cost that the CB incurs as a result of facing uncertainty in the parameter structure of the model. In evaluating policy, expectations are treated as parametric (Currie and Levine 1999) and the time-consistent discretionary solution reduces the problem to a period-by-period optimization of the loss function (3) or (4) subject to (1). We compare next how the existence of such uncertainty affects the results. We optimize thus respectively (3) and (4). Assuming for simplicity that the discount factor is equal to one, i.e. $\beta = 1^2$, the *structural* representation of the results with **Certainty** (when α is treated as a parameter) is then:

$$y_t^{CE} = \frac{\alpha}{1 + \alpha^2} \pi^* - \frac{\alpha}{1 + \alpha^2} E_t \pi_{t+1} - \frac{\alpha}{1 + \alpha^2} \varepsilon_t, \quad \forall t \quad (5)$$

$$\pi_t^{CE} = \frac{\alpha^2}{1 + \alpha^2} \pi^* + \frac{1}{1 + \alpha^2} E_t \pi_{t+1} + \frac{1}{1 + \alpha^2} \varepsilon_t, \quad \forall t \quad (6)$$

¹Note, that uncertainty in α is qualitatively equivalent to uncertainty in γ when one wishes to examine the impact of supply shocks. Walsh 2003 shows that if the central bank had an interest rate term in its objective function and examined the effect of a quasi-demand shock (real interest rate shock), then the uncertainty in either α or γ would have qualitatively different implications for policy implementation. Other attempts which deal with similar types of uncertainty are Ellison and Valla (2001), Sack (2000) and Söderström (2002).

²See Appendix A for the general solution of what follows when $0 < \beta < 1$.

By contrast, the results under **Brainard Uncertainty** are:

$$y_t^{BR} = \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \pi^* - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} E_t \pi_{t+1} - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \varepsilon_t, \quad \forall t \quad (7)$$

$$\pi_t^{BR} = \frac{\bar{\alpha}^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \pi^* + \frac{1 + \sigma_\alpha^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} E_t \pi_{t+1} + \frac{1 + \sigma_\alpha^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \varepsilon_t, \quad \forall t \quad (8)$$

Proposition 1 : *The presence of uncertainty in the transmission process increases the relative contribution of expectations to the final inflation outcome and reduces that of policy.*

Proof 1: We observe that the existence of parameter uncertainty implies the following things:

- As uncertainty increases ($\sigma_\alpha^2 \rightarrow \infty$), the instrument (y_t) is used less and less, in line with Brainard's classical attenuation effect, [from (7)].
- As a consequence, the relative contribution of policy to the inflation outcome (coefficient of π^* in (8)) reduces in uncertainty.
- By contrast, that of expectations (coefficient of $E_t \pi_{t+1}$ in (8)) increases in the level of prevailing uncertainty. At the limit when uncertainty is infinite ($\sigma_\alpha^2 \rightarrow \infty$), it is straightforward to show that the target of the Central Bank becomes irrelevant and all that matters is private sector expectations (naturally shocks always play a role)³.

The unfortunate characteristic of the Brainard attenuation effect is that unless expectations are exactly on target (i.e. $E_t \pi_{t+1} = \pi^*$), the CB will not reach its inflation objective. Two questions follow: first, can the Central Bank safely assume that expectations are always on target? Second, if not, then how does this affect its credibility? In practice inflation targeting countries have a very clear incentive to achieve their inflation objective and worry about the detrimental effects on their ability to achieve their objectives in the future, if they do not. The model used however, does not allow for a feedback from inflation performance to expectations formation and back. To emphasize how important expectations

³The period-by-period objective function $L = \frac{1}{2} [(\bar{\pi} - \pi^*)^2 + y_t^2 (1 + \sigma_\alpha^2)]$ is composed of the first and second moments of the inflation distribution. The first term is thus the square deviation of the average policy from the target, and is minimized for the same value of the instrument that optimizes the loss function under certainty. The second term on the other hand, is a function of the variation of the policy instrument and is minimized when the instrument is not used at all (i.e. $y = 0$). The optimal instrument value that minimizes both terms together, depends on the value of the coefficient of variation, $CV \equiv \frac{\sigma_\alpha}{\bar{\alpha}}$. In other words, the relative importance of the two first moments will decide where the optimal y will be with respect to the two limit values (Onatski 2000). Optimal though this is in terms of welfare it has the somewhat paradoxical implication that the Central Bank is less and less able to achieve its objective, a fact that goes against the principle of inflation targeting. It is our view, therefore, that this set-up is not appropriate to capture the objectives of a central bank that aims to achieve a certain inflation target.

are in determining the final outcome, we will work with equations (7) and (8) which are the *structural* representations of model's solutions in the sense that expectations are not replaced (Leitemo, 2006). The points raised above therefore, are independent of what expectations look like. Svensson (2003) argues in favour of such a representation in order to indicate that factors like judgement, which contribute to the way expectations are formed but cannot always be modelled, are an important contributor to monetary policy. We will thus work with this representation and examine how we can revert this detrimental effect of uncertainty on the Central Bank's ability to achieve its objective, before we solve for expectations. We do this next and discuss further down how to get to the *reduced* form solutions.

3 Two-Step Inflation Targeting

We have thus shown so far that the presence of Brainard uncertainty limits and ultimately immobilizes policy. In turn, this also implies that the inflation objective becomes more and more difficult to achieve, which lends itself to the question of what use is a quantitative objective if it is seldom achieved. Our objective as a consequence, is to identify an algorithm that, given the set-up assumed, reactivates the relevance of policy and aims explicitly at attaining the target. Achieving that, we can then evaluate whether and when such an algorithm actually does better than the Brainard solution shown above and why.

We derive an algorithm based on the following rationale. We know from (8) that the attenuated policy will close only part of the distance between expectations $E_t \pi_{t+1}$ and the target π^* , say $k(\pi^* - E_t \pi_{t+1})$, (where $k < 1$ and reducing in the level of prevailing uncertainty σ_α^2). Closing the full distance requires then that one aims at a target further away from the desired level, say $(\pi^* + \theta)$, such that $k[(\pi^* + \theta) - E_t \pi_{t+1}]$ is the distance covered by the attenuated policy rule that will land at precisely π^* . We do this in two steps: we first derive a policy rule as a function of the overshooting target $(\pi^* + \theta)$ and in the second step then identify the value of θ that matches the degree of "overshooting" to the proportion of the distance that needs to be covered. The rule therefore, aims to hit the target not by aiming directly 'at' it, but aiming 'for' it.

3.1 An optimization framework

We derive the two-step procedure in detail next, and show how it reduces the attenuation effect of Brainard uncertainty.

STEP 1

In the first step, and after the shock has occurred, the monetary policy authority identifies the optimal policy rule as a function of an auxiliary target $(\pi^* + \theta)$. Formally this means optimizing the following objective function instead of (4):

$$\min_y E(L) = \frac{1}{2} E_t \sum_{\tau=0}^{\infty} \beta^\tau \left\{ [\bar{\pi}_t - (\pi^* + \theta)]^2 + y_t^2 (1 + \sigma_\alpha^2) \right\} \quad (9)$$

subject to the supply curve (1). The optimal rule derived following the period-by-period optimization of (9) is a function of θ :

$$\hat{y}_t = \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} (\pi^* + \theta) - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} (E_t \pi_{t+1} + \varepsilon_t) \quad (10)$$

and therefore inflation is:

$$\hat{\pi}_t = \frac{\bar{\alpha}^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} (\pi^* + \theta) + \frac{1 + \sigma_\alpha^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} (E_t \pi_{t+1} + \varepsilon_t) \quad (11)$$

The above two equations imply that for a given level of uncertainty, the CB will choose to deviate, at first instance, from its ultimate target π^* by a parameter θ .

STEP 2

The CB now chooses θ in full knowledge of the extent of uncertainty and the size of the shock and aims to maximize the probability of achieving its true objectives. In other words, since inflation expectations move away from the target as uncertainty increases, the deviation term θ will move to close that gap. In that respect θ is therefore, an auxiliary step, necessary for the Bank to make full use of the information it has. The derived rules from Step 1, (10) and (11), are now inserted into the objective function of the Central Bank:

$$\min_{\theta} E(L) = \frac{1}{2} E_t \sum_{\tau=0}^{\infty} \beta^\tau \left\{ (\hat{\pi}_t - \pi^*)^2 + \hat{y}_t^2 \right\} \quad (12)$$

to produce

$$\min_{\theta} E(L) = f(\theta, \sigma_\alpha^2, \hat{y}_t, \hat{\pi}_t) \quad (13)$$

Given the rule derived in Step 1, the CB chooses now the degree of overshooting θ required, (contingent on the economy's past history and the perceived uncertainty of the transmission of policies), that will get it to π^* , i.e.:

$$\theta(\sigma_\alpha^2) = \arg \min_{\theta} E(L)$$

which in its analytical form is

$$\theta = \frac{\sigma_\alpha^2}{1 + \bar{\alpha}} [\pi^* - E_t \pi_{t+1} - \varepsilon_t] \quad (14)$$

As uncertainty decreases, the deviations (overshooting) from π^* decrease as well, such that at the limit they become zero, i.e.

$$\lim_{\sigma_\alpha^2 \rightarrow 0} (\theta) = 0$$

Proposition 2 : *Applying a two-step procedure in which θ is contingent on the shocks that hit the economy, the existing uncertainty and the inflation target, neutralizes the ex ante effects of uncertainty on the policy rules.*

Proof 2: Substituting the analytical solutions for θ (14) into (10) and (11) produces the *two-step* target rules that a Central Bank needs to apply under uncertainty.

$$y_t^{TS} = \frac{\bar{\alpha}}{1 + \bar{\alpha}^2} \pi^* - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2} (E_t \pi_{t+1} + \varepsilon_t) \quad (15)$$

$$\pi_t^{TS} = \frac{\bar{\alpha}^2}{1 + \bar{\alpha}^2} \pi^* + \frac{1}{1 + \bar{\alpha}^2} (E_t \pi_{t+1} + \varepsilon_t) \quad (16)$$

The rules achieved are similar to those attained with no uncertainty, (24) and (25) but for α being replaced by $\bar{\alpha}$. This demonstrates that varying the target optimally⁴ neutralizes the presence of uncertainty in the transmission process. It is no longer the case that as uncertainty increases, the contribution of expectations to the final inflation outcome increases. This however, happens at the expense of using y_t more actively, thereby introducing greater variability in the system. The question therefore, will be whether for a given level of expectations, the benefits of hitting the target outweigh the costs of greater volatility introduced. We discuss this next.

4 The merits of the TS procedure

Given the problem examined, the Brainard rule is the optimal solution attained. In what sense then is the TS rule useful? In order to answer this we first need to discuss how expectations are formed and thus derive the *reduced* form solutions.

4.1 The Formation of Expectations

The most obvious way of deriving the *reduced* form solutions would be to apply rational expectations through forward recursive substitutions. Respectively for the three cases, the inflation outcome would then be:

$$\tilde{\pi}_t^{CE} = \pi^* + \frac{1}{1 + \alpha^2 - \rho} \varepsilon_t \quad (17)$$

$$\tilde{\pi}_t^{BR} = \pi^* + \frac{1 + \sigma_\alpha^2}{\bar{\alpha}^2 + (1 + \sigma_\alpha^2)(1 - \rho)} \varepsilon_t \quad (18)$$

$$\tilde{\pi}_t^{TS} = \pi^* + \frac{1}{1 + \bar{\alpha}^2 - \rho} \varepsilon_t \quad (19)$$

⁴The word ‘optimally’ here refers to the way uncertainty is accounted for.

Equations (17)-(19) show that when solving for expectations under the RE assumption, the existence of multiplicative uncertainty does not prevent the Central Bank from achieving its objective π^* (but for the supply error and its persistence). The Central Bank attains its inflation objective and the Brainard rule imposes the optimal way of phasing out errors given their degree of persistence. Imposing rational expectations in a standard way ensures therefore, that expectations $E_t\pi_{t+1}$ act as a ‘jump’ variable that always moves to compensate for any shortcomings in the policy action and bring inflation in line with the objective. But this also obfuscates (eliminates) the effect of uncertainty on the relative importance of expectations, evident in the *structural* form.

Alternatively, one could assume an explicit functional form for expectations, as for example Leitemo (2006), who imposes a backward looking, learning type of process for expectations in the model. If the exact way that expectations are formed is known, then the Central Bank needs to start again by optimizing its objective function, subject to the new equations for (1) and (2) that incorporate the assumed expectations formation process. The intertemporal optimization would account for this and would effectively reduce the model to a backward-looking process, where Brainard’s attenuation policy remains the optimal strategy.

However, our motivation for looking for a new algorithm stems from the distinction expressed earlier between the *structural* and *reduced* form representations. The advantage of working with the *structural* representation is that it recognizes that it is not always possible for the CB to know how expectations are formed, and that therefore, by inserting an explicit expectations process in (1) and (2) right at the start, one might be ignoring relevant information that is not easy to model. In other words, the CB needs to work within a framework in which there is not only uncertainty in the transmission process, but also uncertainty in the way that expectations are formed. Our analysis then shows that uncertainty in the transmission implies that those unknown expectations receive a greater weight. The TS algorithm we introduce reduces that effect and effectively neutralizes the outcome from variations in uncertainty.

Faced with undefined expectations, we then carry out the following experiment: suppose the CB operates under the assumption that expectations are rational and applies rule (8). However, having applied this rule, it then turns out that expectations were naive, $E_t\pi_{t+1} = \pi_{t-1}$, instead. Given this error in estimating expectations, would the CB have been better-off applying rule (16) instead? In other words, if expectations turn out to be different to what the CB has based its decision on, is it then not better to have assigned a smaller weight to them? If yes, is this always true or is it a function of the levels of uncertainty prevailing? Note that this analysis no longer gives an ‘optimal’ rule, in the sense of the best the CB can do. Its comparative nature however, describes the robustness of Brainard’s attenuation result in situations where expectations are not automatically ‘rational’.

Expectations do not necessarily need to be backward looking for this question to be interesting. In practice, one expects that there are both forward as well as backward looking agents ('rule of thumb' consumers). The relevant points for our argument is that expectations are not always in line with the Central Bank's objective and that the Central Bank itself is not always confident of what expectations are equal to at the time it takes the interest rate decision. But how appropriate is our assumption of backward looking expectations? In a recent paper, Morris and Shin (2006) argue that it takes only a very small number of 'naive' agents for the whole system to behave as though it was entirely backward looking. They introduce the concept of *Differential Information*, according to which the majority of agents are forward looking (sophisticated) and only a very small proportion is backward looking (naive). They also assume that this proportion of naive agents is very small to start with and actually reduces in size from one period to the next as agents learn. The system therefore, eventually catches up in terms of sophistication. However, if this learning is a slow process, Morris and Shin show that the incentive to coordinate that prevails leads all agents to the second-best outcome in which every one 'imitates' the naive agents. They thus show that the existence of just a small backward looking minority suffices for inertia to be introduced in the system. We use their argument to justify our choice of backward looking expectations as an alternative to forward looking rational expectations, but show how this works in detail in Appendix C.

4.2 When is TS better than BR?

What we examine next is, given that expectations turn out to be backward-looking, what losses does the Central Bank suffer as a result of applying either of the two rules? The dynamic nature of the adjustment to the shock that occurs in the first period implies that multiple periods are required before the system converges to the objective. It is important therefore to calculate the cumulative losses as an estimate of the total cost (losses) associated with eliminating the shock. Starting from given level of inflation, the TS algorithm will introduce greater variability in the system. However, as it gets closer to the target from the second period onwards, the benefits of being closer to the target can potentially outweigh the variability costs such that on aggregate and for small levels of uncertainty the TS algorithm does better. We show this next.

Static losses are evaluated based on:

$$L = \frac{1}{2} E \left\{ (\bar{\pi}_t - \pi^*)^2 + y_t^2 (1 + \sigma_\alpha^2) \right\}$$

Proposition 3 : For $t = 1$, the TS algorithm introduces variability in the system without any benefits in terms of coming closer to the target.

Proof 3: For $t = 1$ expectations are $E_t \pi_{t+1} = \pi_{t-1} = \pi_0$. We substitute the Brainard and TS solutions, (7)-(8) and (15)-(16) respectively in the objective function, to calculate *ex ante* losses. For any given shock ε_t these are:

$$L_{BR,1} = \frac{(1 + \sigma_\alpha^2) [\varepsilon_t + \pi_0 - \pi^*]^2}{2(1 + \bar{\alpha}^2 + \sigma_\alpha^2)}$$

$$L_{TS,1} = \frac{[1 + \bar{\alpha}^2 (1 + \sigma_\alpha^2)] [\varepsilon_t + \pi_0 - \pi^*]^2}{2(1 + \bar{\alpha}^2)^2}$$

It is then straightforward to show that for $\sigma_\alpha^2 > 0$ Brainard losses are always smaller than TS losses, ($L_{BR,1} < L_{TS,1}$),

$$\frac{(1 + \sigma_\alpha^2) [\varepsilon_t + \pi_0 - \pi^*]^2}{2(1 + \bar{\alpha}^2 + \sigma_\alpha^2)} < \frac{[1 + \bar{\alpha}^2 (1 + \sigma_\alpha^2)] [\varepsilon_t + \pi_0 - \pi^*]^2}{2(1 + \bar{\alpha}^2)^2}$$

$$\bar{\alpha}^2 < \bar{\alpha}^2 + \sigma_\alpha^2 \quad (20)$$

Formulation (20) is always true.

Proposition 4 : *For $t \geq 2$ the TS algorithm brings inflation closer to the target π^* and can, under certain circumstances, deliver lower period-by-period and by consequence also cumulative, losses.*

Proof 4: At the end of the first period, the two algorithms would have achieved different levels of inflation, namely π_1^{BR} and π_1^{TS} . As TS is more aggressive it will have managed to close more of the distance between actual inflation and the target, or in other words, $\pi_1^{TS} - \pi^* < \pi_1^{BR} - \pi^*$. By consequence, expectations formed in the second period after the shock has occurred i.e. $t = 2$, will be π_1^{BR} and π_1^{TS} respectively. In turn it is straight forward to show that $[\varepsilon_t + \pi_1^{TS} - \pi^*]^2 < [\varepsilon_t + \pi_1^{BR} - \pi^*]^2$ and that Brainard losses for $t = 2$ can be worse if the following holds:

$$\frac{(1 + \sigma_\alpha^2) [\varepsilon_t + \pi_1^{BR} - \pi^*]^2}{2(1 + \bar{\alpha}^2 + \sigma_\alpha^2)} > \frac{[1 + \bar{\alpha}^2 (1 + \sigma_\alpha^2)] [\varepsilon_t + \pi_1^{TS} - \pi^*]^2}{2(1 + \bar{\alpha}^2)^2}$$

$$\frac{(1 + \sigma_\alpha^2) (1 + \bar{\alpha}^2)^2}{(1 + \bar{\alpha}^2 + \sigma_\alpha^2) [1 + \bar{\alpha}^2 (1 + \sigma_\alpha^2)]} > \frac{\check{\pi}_1^{TS}}{\check{\pi}_1^{BR}}$$

where $\check{\pi}_1^{TS} = [\varepsilon_t + \pi_1^{TS} - \pi^*]^2$ and $\check{\pi}_1^{BR} = [\varepsilon_t + \pi_1^{BR} - \pi^*]^2$. In general, for any period n this condition is

$$\frac{(1 + \sigma_\alpha^2) (1 + \bar{\alpha}^2)^2}{(1 + \bar{\alpha}^2 + \sigma_\alpha^2) [1 + \bar{\alpha}^2 (1 + \sigma_\alpha^2)]} > \frac{\check{\pi}_{n-1}^{TS}}{\check{\pi}_{n-1}^{BR}} \quad (21)$$

We demonstrate through simulations for which values for the coefficient of variation this is true and then compare the cumulative losses implied by the two methods.

4.3 Numerical Simulations: An Example

We now show how Brainard and TS losses compare numerically. We substitute the solutions for output for the two different regimes Brainard (BR) and Two-Step (TS)

$$\begin{aligned} y_t^{BR} &= \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \pi^* - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} E_t \pi_{t+1} - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \varepsilon_t \\ y_t^{TS} &= \frac{\bar{\alpha}}{1 + \bar{\alpha}^2} \pi^* - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2} E_t \pi_{t+1} - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2} \varepsilon_t \end{aligned}$$

in the equation for prices:

$$\pi_t^j = \beta E_t \pi_{t+1} + \alpha_i y_t^j + \varepsilon_t \quad j = BR, TS$$

where parameter α_i is drawn from a distribution $N(\bar{\alpha}, \sigma_\alpha^2)$. Further, expectations are backward looking and errors exhibit a certain degree of persistence, i.e.:

$$\begin{aligned} E_t \pi_{t+1} &= \pi_{t-1} \\ \varepsilon_t &= \rho \varepsilon_{t-1} + v_t \quad v_t \simeq N(0, 1) \end{aligned} \tag{22}$$

Both α_i as well as ε_t are drawn from their respective distributions in the first period. The persistence in the errors implies that it will take multiple periods however, for the system to converge to targets. We calculate losses period-by-period for the two rules respectively:

$$\begin{aligned} L_{BR,t} &= \frac{1}{2} \left\{ (\pi_t^{BR} - \pi^*)^2 + y_{BR,t}^2 \right\} \\ L_{TS,t} &= \frac{1}{2} \left\{ (\pi_t^{TS} - \pi^*)^2 + y_{TS,t}^2 \right\} \end{aligned}$$

For a given number of years, (n), the cumulative losses are discounted by factor, β , i.e.:

$$\sum_{t=1}^n \beta^t L_{j,t} \quad \forall \quad j = BR, TS \quad \text{and} \quad n = 10 \tag{23}$$

We apply the following parameterization⁵:

⁵Note that for $\beta < 1$ then the rules become

$$\begin{aligned} y_t^{BR} &= \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \pi^* - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \beta E_t \pi_{t+1} - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \beta \varepsilon_t \\ y_t^{TS} &= \frac{\bar{\alpha}}{1 + \bar{\alpha}^2} \pi^* - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2} \beta E_t \pi_{t+1} - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2} \beta \varepsilon_t \end{aligned}$$

as shown in Appendix A.

$$\begin{aligned}\beta &= 0.99 \\ \alpha &\simeq N(0.5, \sigma_\alpha^2) \\ \rho &= 0.8 \\ \pi^* &= 1, \quad \pi_0 = 0\end{aligned}$$

The average value of α applied is somewhat higher than what exists in the literature, where it appears to range from a minimum of 0.024 in Woodford (1999) to a maximum of 0.3 in McCallum and Nelson (1999). However, as the qualitative nature of the results is dependent only on the coefficient of variation of α ($CV \equiv \frac{\sigma_\alpha}{\alpha}$) and not just the mean, the choice of numerical value for α is done purely for presentation purposes. The updating of expectations in equation (22) is consistent with Morris and Shin (2006) definition of expectations inertia. The model is similar to the model used by Svensson (1999) and Söderstrom (2002), although the timing of policy responses and effectiveness is different. In the model applied, the policy response is contemporaneous to the supply shock and to the realization of inflation. The lag response of the system to the policy action observed is due to the inertia in expectations formation imposed and there is no built-in lag in the monetary transmission mechanism. As the inflation target is assumed to be different to the starting level for inflation, the economy is subjected both to a supply shock ε_t as well as a policy shift from π_0 to π^* in the first period. Note that the numerical value of the target does not influence the qualitative nature of the results. We then apply the two rules and calculate the impulse response functions for y and π . Cumulative losses for ten periods are calculated in deviation from the targets. Before presenting a detailed welfare analysis, figures (1) and (2) show a typical path for output and inflation produced by the simulations.

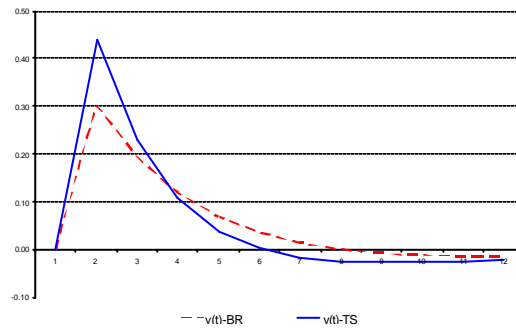


Figure 1: Output Gap - Typical Path

Figure 1 shows a typical path of y , the instrument in our two alternative rules. To achieve the inflation target the economy is subjected to higher real variability

in the early periods of the policy plan, when applying the *two-step* rule. As can be seen in figure 2, once inflation and inflation expectations converge towards the target, the *two-step* policy rule produces both lower real variability as well as a path of inflation closer to the target, relative to the cautious Brainard policy rule.

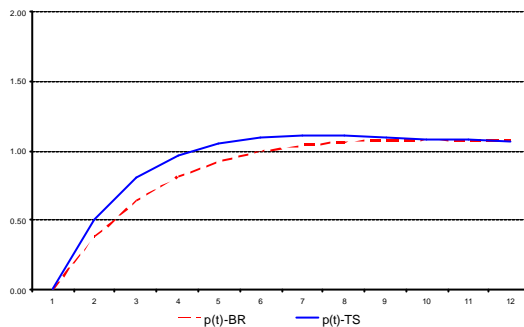


Figure 2: Inflation - Typical Path

Table 2 shows the first period losses of 10,000 stochastic simulations for the two regimes for different degrees of uncertainty captured by the coefficient of variation. In order to stabilize the system around the target, the two-step regime introduces greater variability in the early periods, thus increasing early losses.

CV	$L_{BR}(*10)$	$L_{TS}(*10)$
0.5	11.9	12.1
1	11.6	12.2
1.5	11.4	12.7

Table 3 presents then the average cumulative losses. Cumulative losses are lower in the TS regime for coefficient of variations equal to 0.5 and 1. For higher levels of uncertainty than that, the gains in convergence to the target no longer compensate for the early losses, and Brainard's relative cautious approach reverts to being preferred.

CV	$L_{BR}(*10)$	$L_{TS}(*10)$
0.5	260.7	252.7
1	316.6	292.1
1.5	573.1	1032.0

It is the case therefore, that when evaluating the benefits of two regimes in terms of their dynamic properties, there exist levels of uncertainty for which

it is better to ignore it and simply aim to achieve the objectives set. The variability introduced as result is more than compensated by the benefits of achieving them.

5 Conclusions

Our motivation stems from the observation that the role of policy is reduced in the presence of multiplicative uncertainty and that of expectations is increased. If there are reasons to believe that inflation expectations cannot automatically adjust to the levels the Bank intends, then the presence of Brainard uncertainty can seriously hinder the ability of the Central Bank to achieve its objectives. Our intention then is to find a way of reducing the detrimental effect of uncertainty on the Central Banks' ability to get to its objectives. To demonstrate our point we apply an example of backward looking expectations and motivate our choice through the existence of differential information as put forward by Morris and Shin (2006). Their model presents a system characterized by inertia which, following our analysis, is only worsened in the presence of multiplicative uncertainty. We identify then a two-step algorithm that aims to reintroduce the relevance of policy. This has the advantage that the Central Bank is able to achieve its objectives quicker, but at the expense of introducing greater variability in the system. Our simulations section then shows that in a one-period framework, Brainard does indeed better on average. However, as any shock requires multiple periods before it peters out, we evaluate the cumulative benefits of the two algorithms. We thus show that as the TS algorithm attains the targets quicker, then when uncertainty is relatively low, the benefits of hitting the "bull's eye" outweigh the costs of greater variability. When uncertainty however is relatively large, then Brainard's recommendation for caution carries through.

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APPENDICES

APPENDICES A AND B FOR REFEREEING PURPOSES ONLY

A General Solutions

We derive here the general solutions with the discount factor.

$$L = \frac{1}{2} E \left\{ (\beta E_t \pi_{t+1} + \alpha y_t + \varepsilon_t - \pi^*)^2 + y_t^2 \right\}$$

The discretionary solution of the problem is therefore the following:

$$\begin{aligned} \frac{\partial L}{\partial y} &= \alpha (\beta E_t \pi_{t+1} + \alpha y_t + \varepsilon_t - \pi^*) + y_t = 0 \\ y_t (1 + \alpha^2) &= -\alpha (\beta E_t \pi_{t+1} + \varepsilon_t - \pi^*) \\ y_t &= \frac{\alpha}{1 + \alpha^2} \pi^* - \frac{\alpha}{1 + \alpha^2} (\beta E_t \pi_{t+1} + \varepsilon_t) \end{aligned} \quad (24)$$

Substituting (24) in (1), we obtain the discretionary level of inflation:

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \alpha \left[\frac{\alpha}{1 + \alpha^2} \pi^* - \frac{\alpha}{1 + \alpha^2} (\beta E_t \pi_{t+1} + \varepsilon_t) \right] + \varepsilon_t \\ &= \frac{\alpha^2}{1 + \alpha^2} \pi^* + \frac{1 + \alpha^2 - \alpha^2}{1 + \alpha^2} \beta E_t \pi_{t+1} + \frac{1}{1 + \alpha^2} \varepsilon_t \end{aligned}$$

or

$$\pi_t = \frac{\alpha^2}{1 + \alpha^2} \pi^* + \frac{1}{1 + \alpha^2} \beta E_t \pi_{t+1} + \frac{1}{1 + \alpha^2} \varepsilon_t \quad (25)$$

This equation can be solved forward to obtain (under $\varepsilon_{t+1} = \rho \varepsilon_t$ and calling $\frac{1}{1 + \alpha^2} = A$) a solution for inflation

$$\begin{aligned} \pi_t &= A \alpha^2 \pi^* + A \beta E_t \pi_{t+1} + A \varepsilon_t \\ \pi_{t+1} &= A \alpha^2 \pi^* + A \beta E_t \pi_{t+2} + A \rho \varepsilon_t \end{aligned}$$

then it follows that

$$\begin{aligned} \pi_t &= A \alpha^2 \pi^* + A \beta (A \alpha^2 \pi^* + A \beta E_t \pi_{t+2} + A \rho \varepsilon_t) + A \varepsilon_t \\ &= A \alpha^2 [1 + \beta A + \beta^2 A^2 + \dots] \pi^* + A \varepsilon_t [1 + A \beta \rho + A^2 \beta^2 \rho^2 + \dots] \end{aligned}$$

The two geometric series inside the square brackets are respectively equal to:

$$1 + A\beta + A^2\beta^2 + \dots = \frac{1}{1-A}$$

$$1 + A\beta\rho + A^2\beta^2\rho^2 + \dots = \frac{1}{1-A\rho}$$

Therefore, equilibrium inflation is equal to

$$\pi_t = \frac{A\alpha^2}{1-A\beta}\pi^* + \frac{A}{1-A\beta\rho}\varepsilon_t$$

Substituting for $A = \frac{1}{1+\alpha^2}$, we obtain

$$\begin{aligned}\pi_t &= \frac{\frac{1}{1+\alpha^2}\alpha^2}{1 - \frac{1}{1+\alpha^2}\beta}\pi^* + \frac{\frac{1}{1+\alpha^2}}{1 - \frac{1}{1+\alpha^2}\beta\rho}\varepsilon_t \\ &= \frac{\frac{\alpha^2}{1+\alpha^2}}{\frac{1+\alpha^2-\beta}{1+\alpha^2}}\pi^* + \frac{\frac{1}{1+\alpha^2}}{\frac{1+\alpha^2-\beta\rho}{1+\alpha^2}}\varepsilon_t\end{aligned}$$

and therefore,

$$\pi_t = \frac{\alpha^2}{1+\alpha^2-\beta}\pi^* + \frac{1}{1+\alpha^2-\beta\rho}\varepsilon_t \quad (26)$$

Note that for $\beta = 1$ the result collapse to (??). Similarly under uncertainty when the objective function is now

$$L = \frac{1}{2}E\left\{(\beta E_t\pi_{t+1} + \bar{\alpha}y_t + \varepsilon_t - \pi^*)^2 + y_t^2(1 + \sigma_\alpha^2)\right\}$$

Like above, we solve under the AS constraint only and then identify the i that is implied by the aggregate demand curve. The FOC is:

$$\begin{aligned}\frac{\partial L}{\partial y} &= \bar{\alpha}(\beta E_t\pi_{t+1} + \bar{\alpha}y_t + \varepsilon_t - \pi^*) + y_t(1 + \sigma_\alpha^2) = 0 \\ y_t(1 + \sigma_\alpha^2 + \bar{\alpha}^2) &= -\bar{\alpha}(\beta E_t\pi_{t+1} + \varepsilon_t - \pi^*) \\ y_t &= \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma_\alpha^2}\pi^* - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma_\alpha^2}(\beta E_t\pi_{t+1} + \varepsilon_t)\end{aligned} \quad (27)$$

Substituting (27) in (1), we obtain the discretionary level of inflation:

$$\begin{aligned}\pi_t &= \beta E_t\pi_{t+1} + \bar{\alpha}\left[\frac{\bar{\alpha}}{1 + \bar{\alpha} + \sigma_\alpha^2}\pi^* - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma_\alpha^2}(\beta E_t\pi_{t+1} + \varepsilon_t)\right] + \varepsilon_t \\ &= \frac{\bar{\alpha}^2}{1 + \bar{\alpha} + \sigma_\alpha^2}\pi^* + \frac{1 + \bar{\alpha}^2 + \sigma_\alpha^2 - \bar{\alpha}^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2}\beta E_t\pi_{t+1} + \frac{1 + \bar{\alpha}^2 + \sigma_\alpha^2 - \bar{\alpha}^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2}\varepsilon_t\end{aligned}$$

and therefore,

$$\pi_t = \frac{\bar{\alpha}^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \pi^* + \frac{1 + \sigma_\alpha^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \beta E_t \pi_{t+1} + \frac{1 + \sigma_\alpha^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \varepsilon_t \quad (28)$$

To solve for expectations, under the assumption of rational expectations, we iterate the equation forward (and assume just like above that $\varepsilon_{t+1} = \rho \varepsilon_t$ and calling $\frac{1}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} = \Psi$)

$$\begin{aligned} \pi_t &= \Psi \bar{\alpha}^2 \pi^* + \Psi (1 + \sigma_\alpha^2) \beta E_t \pi_{t+1} + \Psi (1 + \sigma_\alpha^2) \varepsilon_t \\ \pi_{t+1} &= \Psi \bar{\alpha}^2 \pi^* + \Psi (1 + \sigma_\alpha^2) \beta E_{t+1} \pi_{t+2} + \Psi (1 + \sigma_\alpha^2) \rho \varepsilon_t \end{aligned}$$

then we can substitute and iterate forward

$$\begin{aligned} \pi_t &= \Psi \bar{\alpha}^2 \pi^* + (1 + \sigma_\alpha^2) \Psi \beta [\Psi \bar{\alpha}^2 \pi^* + \Psi (1 + \sigma_\alpha^2) \beta E_t \pi_{t+2} + \Psi (1 + \sigma_\alpha^2) \rho \varepsilon_t] + \Psi (1 + \sigma_\alpha^2) \varepsilon_t \\ \pi_t &= \Psi \bar{\alpha}^2 \left[1 + (1 + \sigma_\alpha^2) \beta \Psi + (1 + \sigma_\alpha^2)^2 \beta^2 \Psi^2 \dots \right] \pi^* + \Psi (1 + \sigma_\alpha^2) \left[1 + \Psi \beta (1 + \sigma_\alpha^2) \rho + \Psi^2 \beta^2 (1 + \sigma_\alpha^2)^2 \rho^2 \dots \right] \varepsilon_t \end{aligned}$$

The two geometric series inside quadratic brackets are equal to

$$\begin{aligned} \left[1 + (1 + \sigma_\alpha^2) \beta \Psi + (1 + \sigma_\alpha^2)^2 \beta^2 \Psi^2 \dots \right] &= \frac{1}{1 - (1 + \sigma_\alpha^2) \beta \Psi} \\ \left[1 + \Psi \beta (1 + \sigma_\alpha^2) \rho + \Psi^2 \beta^2 (1 + \sigma_\alpha^2)^2 \rho^2 \dots \right] &= \frac{1}{1 - \Psi (1 + \sigma_\alpha^2) \beta \rho} \end{aligned}$$

Therefore, equilibrium inflation is equal to

$$\pi_t = \frac{\Psi \bar{\alpha}^2}{1 - (1 + \sigma_\alpha^2) \beta \Psi} \pi^* + \frac{\Psi (1 + \sigma_\alpha^2)}{1 - (1 + \sigma_\alpha^2) \beta \Psi} \varepsilon_t$$

Substituting back the value for $\Psi = \frac{1}{1 + \bar{\alpha}^2 + \sigma_\alpha^2}$ we obtain

$$\begin{aligned} \pi_t &= \frac{\frac{1}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \bar{\alpha}^2}{1 - (1 + \sigma_\alpha^2) \beta \frac{1}{1 + \bar{\alpha}^2 + \sigma_\alpha^2}} \pi^* + \frac{\frac{1}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} (1 + \sigma_\alpha^2)}{1 - \frac{1}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} (1 + \sigma_\alpha^2) \beta \rho} \varepsilon_t \\ &= \frac{\frac{\bar{\alpha}^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2}}{\frac{1 + \bar{\alpha}^2 + \sigma_\alpha^2 - \beta - \beta \sigma_\alpha^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2}} \pi^* + \frac{\frac{(1 + \sigma_\alpha^2)}{1 + \bar{\alpha}^2 + \sigma_\alpha^2}}{\frac{1 + \bar{\alpha}^2 + \sigma_\alpha^2 - (1 + \sigma_\alpha^2) \beta \rho}{1 + \bar{\alpha}^2 + \sigma_\alpha^2}} \varepsilon_t \end{aligned}$$

and therefore,

$$\pi_t = \frac{\bar{\alpha}^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2 - \beta - \beta \sigma_\alpha^2} \pi^* + \frac{1 + \sigma_\alpha^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2 - (1 + \sigma_\alpha^2) \beta \rho} \varepsilon_t \quad (29)$$

In both case we see that the smaller the discount factor (and therefore, the higher the degree of impatience) the longer it takes to come to the target π^* .

B The Loss Function with Uncertainty in α

Under uncertainty, where $\alpha_t \rightarrow N(\bar{\alpha}, \sigma_\alpha^2)$, losses conditional on shocks ε , we can express the objective function of the CB in terms of the moments of α .

$$\begin{aligned}
L &= \frac{1}{2} E \left\{ (\pi_t - \pi^*)^2 + y_t^2 \right\} \\
&= \frac{1}{2} E (\pi_t - \pi^*)^2 + \frac{1}{2} E (y_t^2) \\
&= \frac{1}{2} E \{ E_t \pi_{t+1} + \alpha y_t + \varepsilon_t - \pi^* \}^2 + \frac{1}{2} E (y_t^2) \\
&= \frac{1}{2} E \left\{ (E_t \pi_{t+1} + \varepsilon_t - \pi^*)^2 + (\alpha y_t)^2 \right. \\
&\quad \left. + 2(E_t \pi_{t+1} + \varepsilon_t - \pi^*) (\alpha y_t) \right\} + \frac{1}{2} E (y_t^2) \\
&= \frac{1}{2} \left\{ (E_t \pi_{t+1} + \varepsilon_t - \pi^*)^2 + E (\alpha y_t)^2 \right. \\
&\quad \left. + 2(E_t \pi_{t+1} + \varepsilon_t - \pi^*) E (\alpha y_t) \right\} + \frac{1}{2} E (y_t^2)
\end{aligned}$$

but since $E(\alpha y_t)^2 = y_t^2 E(\alpha)^2$ and $E(\alpha^2) = \sigma_\alpha^2 + \bar{\alpha}^2$ then,

$$\begin{aligned}
L &= \frac{1}{2} \left\{ (E_t \pi_{t+1} + \varepsilon_t - \pi^*)^2 + y_t^2 E(\alpha)^2 \right. \\
&\quad \left. + 2(E_t \pi_{t+1} + \varepsilon_t - \pi^*) \bar{\alpha} y_t \right\} + \frac{1}{2} E (y_t^2) \\
&= \frac{1}{2} \left\{ (E_t \pi_{t+1} + \varepsilon_t - \pi^*)^2 \right. \\
&\quad \left. + y_t^2 (\sigma_\alpha^2 + \bar{\alpha}^2) + 2(E_t \pi_{t+1} + \varepsilon_t - \pi^*) \bar{\alpha} y_t \right\} + \frac{1}{2} E (y_t^2) \\
&= \frac{1}{2} \left\{ (E_t \pi_{t+1} + \varepsilon_t - \pi^*)^2 + y_t^2 \bar{\alpha}^2 \right. \\
&\quad \left. + 2(E_t \pi_{t+1} + \varepsilon_t - \pi^*) \bar{\alpha} y_t + y_t^2 \sigma_\alpha^2 \right\} + \frac{1}{2} E (y_t^2)
\end{aligned}$$

From this, it follows that

$$\bar{\pi}_t \equiv E(\pi_t) = E_t \pi_{t+1} + \bar{\alpha} y_t + \varepsilon_t$$

and therefore,

$$\begin{aligned}
L &= \frac{1}{2} \left\{ (\bar{\pi}_t - \pi^*)^2 + y_t^2 \sigma_\alpha^2 + y_t^2 \right\} \\
&= \frac{1}{2} \left\{ (\bar{\pi}_t - \pi^*)^2 + y_t^2 (\sigma_\alpha^2 + 1) \right\}
\end{aligned}$$

C Introducing Differential Information

The concept of *Differential Information* builds on previous work by Morris and Shin (Morris and Shin, 2002a and 2002b). When applied to monetary policy the idea behind this is as follows: the current state of inflation is π_0 but the Central Bank wants to move it to a new level π^* . Inflation state π at time t evolves in the following way:

$$\pi_t = \begin{cases} \pi_0 & \text{for } t = 0 \\ \pi^* & \text{for } t \geq 1 \end{cases}$$

However, in the eyes of those who form expectations, the assumption of differential information implies that at time $t \geq 1$ only a proportion of the private sector μ knows the value of π_t and automatically adjusts expectations to that level (π^*), whereas, $1 - \mu$ does not. The authors assume that the proportion of people, $(1 - \mu)$, that are effectively "unaware" of this target is very small to start with (μ is very close but not quite one) and diminishes from period to period, such that eventually everyone adjusts their expectation in line with the target. This latter assumption implies that their approach is consistent with Rational Expectations. Moreover, MS assume that μ_1 is very close to one to start with, such that the informational friction is sufficiently small by comparison to the occasion of no differential information. Looking at the agents individually, it is then the case that informed agents form expectations as:

$$E_{i,t} \begin{bmatrix} \pi_0 \\ \pi_{t+h} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix} \quad (30)$$

whereas uninformed agents form expectations as:

$$E_{i,t} \begin{bmatrix} \pi_0 \\ \pi_{t+h} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix} \quad (31)$$

The assumption therefore, is that everyone knows the current or past state of inflation. Then, since a proportion μ_t are informed at time t , average expectation for inflation is

$$\bar{E}_t \begin{bmatrix} \pi_0 \\ \pi_{t+h} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 - \mu_t & \mu_t \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix} \quad (32)$$

Similarly,

$$\begin{aligned}
\bar{E}_{t-1}\bar{E}_t \begin{bmatrix} \pi_0 \\ \pi_{t+h} \end{bmatrix} &= \bar{E}_{t-1} \begin{bmatrix} 1 & 0 \\ 1 - \mu_t & \mu_t \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 \\ 1 - \mu_t & \mu_t \end{bmatrix} \bar{E}_{t-1} \begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 \\ 1 - \mu_t & \mu_t \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 - \mu_{t-1} & \mu_{t-1} \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 \\ 1 - \mu_{t-1}\mu_t & \mu_{t-1}\mu_t \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix}
\end{aligned}$$

In a forward looking world, the present is a function of the sequence of all future expectations. In other words, to derive the appropriate expectation one needs to iterate this forward such that

$$\bar{E}_1\bar{E}_2\dots\bar{E}_t \begin{bmatrix} \pi_0 \\ \pi_{t+h} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 - \prod_{s=1}^t \mu_s & \prod_{s=1}^t \mu_s \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix}$$

and therefore,

$$\bar{E}_1\bar{E}_2\dots\bar{E}_t(\pi_0) = \pi_0 \tag{33}$$

and expected inflation h periods ahead is

$$\bar{E}_1\bar{E}_2\dots\bar{E}_t(\pi_{t+h}) = \left(1 - \prod_{s=1}^t \mu_s\right) \pi_0 + \left(\prod_{s=1}^t \mu_s\right) \pi^* \tag{34}$$

Note that in the absence of differential information ($\mu = 1$) iterated expectations collapse to the single expectation at time t , which under RE is equal to π^* . However for $\mu < 1$, the higher order expectations for a given timing $t + h$ (where $h > 0$) depends on the limiting property of $\prod_{s=1}^t \mu_s$. And if μ_s approaches one only when $t \rightarrow \infty$, then it follows that $\prod_{s=1}^t \mu_s \rightarrow 0$ (even if μ_s is very close to one to start with) and the current level of inflation π_0 prevails in (34). The intuition behind this stems from the fact that monetary policy is an information game between the central bank and the private agents but also between the private agents themselves⁶. The element of coordination between the agents is important in the process of forming expectations where second-guessing how others think is crucial to one's decision. In the presence of differential information, those that are aware of the target are also aware that there is a very small minority that is not and will therefore form expectations according to the current level of inflation. Knowing that, the desire to coordinate "forces" them to match their expectations to those of the least informed group, namely at π_0 . The existence of such small minority of people implies that π^* can therefore never be attained. The effect of such an assumption is

⁶See Demertzis and Viegli (2006) for a detailed description of monetary policy as an information game.

that while the economy is forward-looking, the system effectively operates as though it were backward-looking.

In order to allow for differential information in the context of our model, inflation expectation $E_t\pi_{t+1}$ is now proxied by its higher order equivalent, i.e.:

$$\bar{E}_{t-k}\bar{E}_{t-k+1}\dots\bar{E}_t(\pi_{t+1}) = \left(1 - \prod_{s=t-k}^t \mu_s\right) \pi_0 + \left(\prod_{s=t-k}^t \mu_s\right) \pi^* \quad (35)$$

What (35) then tells us is that the longer it takes for μ to converge to 1 (k is a large number), the closer to the original level of inflation π_0 expectations remain. Substituting then (35) in (8) we can now see that this effect is exacerbated in the presence of uncertainty.

$$\begin{aligned} \pi_t &= \frac{\bar{\alpha}^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \pi^* + \frac{1 + \sigma_\alpha^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \left[\left(1 - \prod_{s=t-k}^t \mu_s\right) \pi_0 + \left(\prod_{s=t-k}^t \mu_s\right) \pi^* \right] \\ &+ \frac{1 + \sigma_\alpha^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \varepsilon_t \\ &= \left[\frac{\bar{\alpha}^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} + \left(\prod_{s=t-k}^t \mu_s\right) \right] \pi^* + \frac{1 + \sigma_\alpha^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \left(1 - \prod_{s=t-k}^t \mu_s\right) \pi_0 \\ &+ \frac{1 + \sigma_\alpha^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \varepsilon_t \end{aligned} \quad (36)$$

In practice (36) implies that there are both backward looking as well as forward looking in the agents and their effect on the system's inertia is determined by the limiting properties of μ . For demonstration purposes only however, we will consider the limit case when $k \rightarrow \infty$ in the main text, or in other words, when agents take a very long time to update their beliefs and expectations are therefore, purely backward-looking:

$$\lim_{k \rightarrow \infty} \bar{E}_{t-k}\bar{E}_{t-k+1}\dots\bar{E}_t(\pi_{t+1}) = \pi_0 \quad (37)$$

Substituting then (37) into (36) produces:

$$\pi_t = \frac{\bar{\alpha}^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \pi^* + \frac{1 + \sigma_\alpha^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \pi_0 + \frac{1 + \sigma_\alpha^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \varepsilon_t \quad (38)$$

This shows that the system exhibits inertia and therefore, the ability of the Central Bank to achieve its objectives is seriously hindered. The inflation outcome will therefore, not close the whole distance between π_0 and π^* . It is then straight forward to illustrate from (38) the role of uncertainty in exacerbating this discrepancy. Even if a little differentiated information is introduced, the existence of uncertainty emphasizes the role of expectations in terms of determining the

outcome and de-emphasizes that of policy. At the limit when uncertainty is infinite, the central bank is unable to move away from the current level of inflation. In other words,

$$\lim_{\sigma_a^2 \rightarrow \infty} \pi_t = \pi_0 + \varepsilon_t \quad (39)$$

This implies that in the presence of uncertainty, it becomes increasingly difficult for policy to achieve its inflation objective and the system is characterized by even greater inertia than is due to just differential information.