The 'Wisdom of the Crowds' and Public Policy*

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Abstract

Surowiecki (2004) argues that collective predictions are better than individual predictions and calls that the *Wisdom of the Crowds*. We use an analytical information model to demonstrate and explain this. Then we see how these two predictions are affected by better public information and show that while individual predictions always improve, collective ones do not. A social planner that relies on collective predictions to form policy may erroneously refrain from providing better information. We use two examples to show where this might be applicable.

Keywords: Public information, social planner, 'expert' vs 'lay' crowds.
JEL codes: D82, E52,E58.

1 Introduction

In his book the *Wisdom of the Crowds*, Surowiecki (2004) claims that "many are smarter than the few". He argues that "...under the right circumstances, groups are remarkably intelligent, and are often smarter than the smartest people in them." We show that the error a crowd makes collectively, when forecasting a state θ (error of average forecast of θ), is smaller than the average error made by the individuals (average of individual forecast errors of θ). We use a small analytical model, in which the individual makes optimal use of all the information she has, to explicitly demonstrate this.
Then we show how the provision of better public information always helps individuals reduce their forecast error. However, we show that while this might be true for the individual, the 'crowd' does not necessarily forecast better as a result of better public information. This depends on how much better its

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own private information is relative to the public information provided. And we show that when private information is relatively better than public information, providing more of the latter simply shifts the emphasis to a relatively inferior set of information and is therefore detrimental to the final output.
But then the question arises as to how relevant is the crowd forecast? And if the individual always benefits from more information, then why does it matter that the crowd forecast does not always benefit from more accurate public information? We argue that the crowd forecast is often publicly available and social planners rely on it to form policy decisions. By implication, there will be occasions for which the social planner may erroneously refrain from providing better information. We discuss this in the context of two examples.

The paper is organized as follows. Section 2 models and verifies the *Wisdom of the Crowds*. Section 3 then explains the impact of better information on the ability to forecast and illustrates the argument with two examples. Section 4 concludes.

2 The 'Wisdom of the Crowds'

Agent i forecasts state θ subject to all information available to her.

Definition 1 *Individual i's forecast (f) of state θ is f_i.*

We assume agent i chooses f to minimize the variance of her forecast error:

$$u_i(\theta) \equiv \mathbb{E}_i(f-\theta)^2. \tag{1}$$

Subscript i in the expectations operator indicates that the individual uses all information at her disposal to make this forecast. Based on (1), the individual's best forecast is:

$$\begin{aligned} f_i &= \arg\min u_i(\theta) \\ &= \mathbb{E}_i(\theta). \end{aligned}$$

Information about θ is available in the form of a public signal common to all agents, and a private signal, specific to each agent in the economy. Information is then summarized as follows[1]:

$$\text{Public signal:} \quad y = \theta + \eta. \tag{2}$$

$$\text{Private signal:} \quad x_i = \theta + \varepsilon_i. \tag{3}$$

Signal errors η and ε_i have variance σ_η^2 and σ_ε^2 respectively (and are non-zero and finite). Furthermore, noise terms ε_i of the continuum population have zero

[1] Based on Morris and Shin (2002).

mean and both error terms are independent of θ and of each other, such that $E(\varepsilon_i \varepsilon_j) = 0$ for $i \neq j$. The best linear unbiased estimator of θ implies that the agent weighs the two types of information by their relative precision, to form a view about the state, i.e.:

$$\begin{aligned} f_i = \mathbb{E}_i(\theta) &= \frac{\alpha y}{\alpha + \beta} + \frac{\beta x_i}{\alpha + \beta} \\ &= \theta + \frac{\alpha \eta + \beta \varepsilon_i}{\alpha + \beta}, \end{aligned} \tag{4}$$

where $\alpha = \frac{1}{\sigma_\eta^2}$ and $\beta = \frac{1}{\sigma_\varepsilon^2}$ are the precision of public and private information respectively. Equation (4) represents the individual's best forecast. The variance of the error she then makes, from (1), is:

$$u_i(\theta) = \frac{1}{\alpha + \beta}. \tag{5}$$

Equation (5) represents also the society forecast error, for a continuum of agents indexed by the unit interval $[0,1]$, $\mathbb{U}(\theta) = \int_0^1 u_j dj$. We refer to them interchangeably. We define next the aggregate forecast produced by the agents:

Definition 2 *The average forecast of θ across all individuals is $\bar{f} = \int_0^1 f_j dj$. We identify this with the forecast of the 'crowd'.*

By definition then, the crowd forecast is equal to:

$$\bar{f} = \theta + \frac{\alpha \eta}{\alpha + \beta}, \tag{6}$$

and the variance of the crowd forecast error is equal to:

$$\mathbb{C}(\theta) = \mathbb{E}\left(\bar{f} - \theta\right)^2 = \frac{\alpha}{(\alpha + \beta)^2}. \tag{7}$$

Claim 3 *"...when our imperfect judgements are aggregated in the right way, our collective intelligence is often excellent.", Surowiecki (2004), (p. XIV).*[2]

[2] Surowiecki, (2004) identifies this with the 'Wisdom of the Crowds' in which he argues that a crowd is wiser than any individual if the following four characteristics define it (see wikipedia):

1. **Diversity of opinion**: Each person should have private information even if it's just an eccentric interpretation of the known facts, (existence of ε_i).
2. **Independence**: People's opinions aren't determined by the opinions of those around them, ($E(\varepsilon_i \varepsilon_j) = 0$ for $i \neq j$).
3. **Decentralization**: People are able to specialize and draw on local knowledge.
4. **Aggregation**: Some mechanism exists for turning private judgments into a collective decision, (aggregation mechanism, $\int_0^1 f_j dj$).

We verify that crowds are wiser, by comparing the variance of the society forecast error (5) to that of the crowd forecast (7).

$$\begin{aligned} \mathbb{C}(\theta) &< \mathbb{U}(\theta) \quad \text{or,} \\ \frac{\alpha}{(\alpha+\beta)^2} &< \frac{1}{\alpha+\beta}. \end{aligned} \tag{8}$$

A crowd's forecast is closer to the state than that of the individual on average, and it is in this sense that the crowd is 'wiser'. As the aggregate forecast error nests the crowd's forecast error, i.e.,

$$\begin{aligned} u_i(\theta) &= \mathbb{E}(f_i - \theta)^2 \\ &= \left(\bar{f} - \theta\right)^2 + \sigma_{f_i}^2 \qquad (9) \\ &= \mathbb{C}(\theta) + \sigma_{f_i}^2, \qquad (10) \end{aligned}$$

we observe that for a heterogenous public (i.e. $\sigma_{f_i}^2 \neq 0$), (8) is always true[3].

3 Information and the Social Planner

A social planner aims to assist agents reduce their forecast error, by providing better public information. The quality of the public information signal α is an instrument in the hands of the social planner and any policy pursued should be based on the way it impacts $\mathbb{U}$.

Proposition 4 *An agent (and society) forecasts better as a result of better information.*

Proof: We calculate the effect of better information on the individual's forecast error, in other words, $\frac{\partial u_i}{\partial \alpha}$ and $\frac{\partial u_i}{\partial \beta}$:

$$\frac{\partial u_i}{\partial \alpha} < 0, \quad \forall \alpha; \qquad \frac{\partial u_i}{\partial \beta} < 0, \quad \forall \beta.$$

Starting from (9) we can then look closer at how information improves the individual's forecast.

Definition 5 *A crowd, whose private information is better than the available public information, i.e. $\beta > \alpha$, is defined as 'expert'. The opposite, i.e. $\beta < \alpha$, defines a 'lay' crowd.*

We re-write (9) as follows:

[3] Through Jensen's inequality.

$$u_i(\theta) = \mathbb{E}\left(\bar{f} - \theta\right)^2 + \sigma_{f_i}^2$$
$$= \frac{\alpha}{(\alpha+\beta)^2} + \frac{\beta}{(\alpha+\beta)^2}. \quad (11)$$

So the welfare of society is characterized by two parts: the first term represents how well the crowd forecasts:

$$\mathbb{C} = \mathbb{E}\left(\bar{f} - \theta\right)^2 = \frac{\alpha}{(\alpha+\beta)^2}.$$

We observe that information does not have the same impact on the crowd forecast as it does on society.

Proposition 6 *The variance of the crowd forecast error reduces as a result of better private information. Public information, on the other hand, is only beneficial when the crowd's private information is relatively inaccurate. 'Expert' crowds will see their forecasts worsen as a result of better public information precision.*

Proof: We observe the following:

1. Better private information is always beneficial: $\frac{\partial \mathbb{C}}{\partial \beta} < 0$, $\forall \beta$.
2. Better public information is beneficial only to a relatively 'lay' public: $\frac{\partial \mathbb{C}}{\partial a} < 0$, when $\alpha > \beta$.

We next move to the second term, the variance of the forecasts, $\sigma_{f_i}^2$.

$$\sigma_{f_i}^2 = \mathbb{E}\left(f_i - \bar{f}\right)^2 = \frac{\beta}{(\alpha+\beta)^2}.$$

Proposition 7 *Agents' forecasts become more homogenous as a result of better public information, but not necessarily as a result of better private information.*

Proof: With relevance to the variance of the prediction, we observe the following:

1. Better public information is always beneficial: $\frac{\partial \sigma_{f_i}^2}{\partial a} < 0$, $\forall \alpha$.
2. Better private information is beneficial only to a relatively 'expert' public: $\frac{\partial \sigma_{f_i}^2}{\partial \beta} < 0$, when $\alpha < \beta$.

Intuitively, these observations can be summarized as follows. By nature of its commonality, improved public information makes people more homogenous, but it does not necessarily help them forecast better on average. It depends on how well their own private information fairs, in terms of precision, to the public

information provided. Better public information will only help them forecast better if they are relatively 'lay', because they would then be shifting emphasis from a relatively poor information set to a relatively precise one. If they are however relatively 'expert', then better public information will cause a shift of emphasis to the relatively bad information set and is therefore detrimental to the average forecast. On the other hand, more accurate private information necessarily puts a crowd in a better position to forecast, effectively increasing its expertise, but could, at the same time, increase the heterogeneity of the crowd. Heterogeneity will only decrease if the crowd is relative expert, in which case further increases in the precision of private information give it, deservedly, more emphasis.

3.1 A Discussion

In this section we discuss two examples to demonstrate the context in which a crowd forecast is both available, as well as the measure policy makers may rely on to take policy decisions.

Consider a health policy example, concerning the daily intake of vitamins. For a number of vitamins, there is an optimal quantity[4] people should take daily (θ), which is not necessarily known to all. Each individual consumes the amount she deems appropriate, f_i, based on the information she has. In monitoring this, the health policy maker observes the average intake per capita ($\bar{f}$), and provides information about the benefits and dangers of vitamins. The 'wisdom of the crowd' argument shows that the average intake is indeed closer to θ than the individual intake is from θ on average. This can be misleading in the following sense. In trying to educate the public, the social planner provides more and better information about θ. As we have explained above, this average intake does not always come closer to θ as a result of better public information. There are circumstances therefore in which the social planner may erroneously decide to withhold information, even though society would unequivocally benefit from more information on the subject.

Our second example comes from the literature on payments systems, and relates to the discussion on the substitutability between alternative means of payment, say cash and debit cards. Every individual has a private monetary threshold f_i below which she pays with cash, and above which she pays with a debit card.[5] At the same time there is a socially optimal threshold, θ, which exploits the cost-benefit structure of different payment methods. It is the social planner's task to inform individuals about this level and to steer them in its direction. The social planner observes an average switching threshold $\bar{f}$, and this is what he aims to align with θ. As explained above, public campaigns that aim to

[4] Excess amounts of fat soluble vitamins, contrary to the water soluble ones, are more difficult to eliminate from the body and can result to toxicity. This implies that there is a maximum intake that should be consumed daily (R.J. Kutsky, 1973).

[5] See Brits and Winder, (2005) for the case of the Netherlands.

achieve this, do not always manage to close the distance between the two, even though individuals always benefit from them.

The literature on the benefits and costs of better public information usually includes Keynes' 'beauty term' (see Amato et al, 2003, Morris and Shin, 2002, 2007, Hellwig, 2004),[6] in which the individual is not only concerned with forecasting the fundamental value of the state, but also with checking how far her prediction is from the average forecast. This generates an incentive to coordinate with others, which as argued by Morris and Shin (2002) implies that individuals pay disproportionately more attention to the public signal than is justified by its precision level. Their analysis showed that more public information is not always beneficial to society, and that indeed the benefits of public information depend on how accurate it is relative to private information. A similar incentive is generated when the state is not exogenous to people's beliefs (i.e. θ is a function of $\bar{f}$), as shown in Angeletos and Pavan, (2007), and for a monetary policy example in Demertzis and Viegi (2008). In this paper we show that when the state is exogenous, society always benefits from public information but the average forecast does not. It is again the precision of public information relative to private information that determines how the average forecast is affected. To the extent that policy makers rely on it to form decisions, society might loose out.[7]

4 Conclusion

In a model where an individual aims to forecast a certain state, the error of the collective forecast is always smaller than the average forecast error. This is the sense in which crowds are wiser. The two however are not affected in the same way by the provision of better public information. Thus, while society always forecasts better as a result of a better public information signal, the crowd does not. It depends on its degree of expertise, or in other words, on the quality of private information it possesses relatively to the public information set. We argue that if social planners rely on the crowd forecast to take policy decisions, they may erroneously refrain from providing better information. In the absence of a beauty term in the individual's objective function, or when the state is

[6] Morris and Shin (2002) have included a beauty term in which the individual is aiming to both forecast the state as well as identify how far she is from everybody else's prediction, i.e.:

$$u_i(\theta) \equiv -(1-r)(f_i-\theta)^2 + r(L_i-\bar{L})^2$$

where

$$L_i = \int_0^1 (f_i - f_j)^2\, dj \text{ and } \bar{L} = \int_0^1 L_j dj.$$

Parameter r represents the 'beauty term'.

[7] Note that acquiring better information is assumed to be costless here. If this assumption is relaxed, and information is either costly to acquire or process, society does not necessarily benefit from unbounded information (see Demertzis and Hoebericths, 2007, for an example).

simply exogenous, a social planner should always reveal better information, if he has it.

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