COMPUTERISED CALCULATION OF FORCE DISTRIBUTIONS
IN BOLTED END PLATE CONNECTIONS ACCORDING TO EUROCODE 3

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ABSTRACT

Eurocode 3 provides a procedure for the determination of the bolt-row force distribution in end plate connections. This procedure is a main component in the determination of the design moment resistance ($M_{Rd}$). In principle, it is aimed at both hand and computer calculations. For computer calculations, however, an alternative method leads to a more efficient algorithm. This paper describes such an alternative method. It is based upon plastic analysis of the joint and is fully compatible with Eurocode 3.

1. INTRODUCTION

Since Eurocode 3 (EUROCODE 3, 1992) is now available as a European Prenorm, interest has been expressed in the development of corresponding code check software. Examples of this software are the programs developed based on the EUREKA "CIMSTEEL" design procedures (BROZETTI, 1991). These programs focus at bolted end plate joints according to Eurocode 3.

Practical experience suggests that direct computer implementation of the Eurocode 3 procedure for plastic force distributions (procedure J.3.1 in Eurocode 3 part 1 annex J) in software is not very elegant and maintainable from a system-developers point of view. This procedure determines the moment resistance ($M_{Rd}$) of a bolted end plate connection based on plastic distribution of bolt forces. The problem arises from the fact that the procedure described in Eurocode 3 must also be used by practitioners. These requirements are not as formal as those needed for good computer software development because practitioners should be able to follow the design process. In computer software, this is not of interest, as long as the results agree with the Eurocode 3 requirements.

This paper presents an alternative formulation for the plastic procedure of Eurocode 3 and discusses the problems related to this formulation. In comparison with the procedure of Eurocode 3, this formulation is efficient for application in software. An example is given to clarify its use.

2. PLASTICITY

Eurocode 3 provides rules for the calculation of the moment resistance of a bolted end plate connection, as for example given in figure 1, based on a plastic force distribution. Plasticity allows to assume any force distribution in the joint as long as all components in the joint are able to resist these forces and the force distribution is in equilibrium with the external load (M in figure 1). The number of feasible force distributions is infinitive. There is only one force distribution, however, that leads to the highest external load (M_{Rd} in figure 1). According to Prager (PRAGER, 1977), all other force distributions will be lower bound solutions to this highest external load.

In other words, the characteristics of the plastic analysis of the bolted end plate joints are:

<table>
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<th>Objective</th>
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<td><strong>The external load acting at the joint should be maximised</strong></td>
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<tr>
<td>with the following constraints</td>
</tr>
<tr>
<td><strong>There should be equilibrium in the joint;</strong></td>
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<tr>
<td><strong>Internal forces should not exceed plastic capacities of joint components;</strong></td>
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Now we will elaborate these characteristics concerning the example given in figure 2. We define three parts in this cruciform joint with asymmetrical loads: a beam-to-column connection on the left hand side (LHS), beam-to-column connection on the right hand side (RHS) and a web shear panel (ECCS, 1992). The shear panel is subjected to interaction between the external loads acting in the LHS and RHS beam. Although the characteristics are explained with the help of the example given in figure 2, the method is generally applicable, also for other joints and loads, as for example shown in figure 1.

3. OBJECTIVE

A joint should be able to resist all realistic loading combinations. We assume that normally a designer checks this for a limited number of controlling load cases. To verify the resistance of the joint for a given loading combination (V_t = 35 kN, N_t = 10 kN, M_t = 40 kNm, V_l = 20 kN, M_l = 18 kNm and V_h = 30 kN, see figure 2), we can adopt a 2 step approach.

1. Determine the internal tensile forces in the bolt-rows (F_{t(i)}Sd for i=1 to 9) and the forces in the points of compression (F_{c(j)}Sd for j=1 to 4), and verify the capacity of the joint for the loading combination (N_t = 10 kN, M_t = 40 kNm, M_l = 18 kNm and V_h = 30 kN) that excludes the shear loading V_t and V_l;
2. Verify the capacity of the joint for shear forces in the beams (V_t = 35 kN, V_l = 20 kN). These shear checks are not considered in this paper because in practice this is a minor problem.

In the first step the objective is to maximise \( \lambda \), see figure 2, under the condition that \( \lambda \leq 1 \). The corresponding value for the maximum \( \lambda \) is equal to \( \lambda_{\text{max}} \). If \( \lambda_{\text{max}} \leq 1 \), the joint has insufficient strength. If \( \lambda_{\text{max}} = 1 \), we turn to step 2.
It is also possible not to restrict to $\lambda \leq 1$. Then $\lambda_{\text{max}}$ indicates the over-strength of the joint for a given loading combination.

4. EQUILIBRIUM

In a cruciform joint we can define a number of equilibrium equations: in the LHS connection, in the RHS connection and in the shear web panel.

1. Moment equilibrium in the LHS connection of the joint, for example:

$$-0.04 \cdot F_t(6) \cdot S_d + 0.06 \cdot F_t(7) \cdot S_d + 0.26 \cdot F_t(8) \cdot S_d - 0.01 \cdot F_c(3) \cdot S_d - 0.31 \cdot F_c(4) \cdot S_d + 18.0 \cdot \lambda = 0$$  

   (1)

2. Horizontal equilibrium in the LHS connection of the joint, for example:

$$F_t(6) \cdot S_d + F_t(7) \cdot S_d + F_t(8) \cdot S_d = 0$$  

   (2)

3. Moment equilibrium in the RHS connection of the joint, for example:

$$0.06 \cdot F_t(1) \cdot S_d - 0.08 \cdot F_t(2) \cdot S_d - 0.19 \cdot F_t(3) \cdot S_d - 0.30 \cdot F_t(4) \cdot S_d - 0.44 \cdot F_t(5) \cdot S_d + 0.01 \cdot F_c(1) \cdot S_d + 0.37 \cdot F_c(2) \cdot S_d + 41.9 \cdot \lambda = 0$$  

   (3)

4. Horizontal equilibrium in the RHS connection of the joint, for example:

$$F_t(1) \cdot S_d + F_t(2) \cdot S_d + F_t(3) \cdot S_d + F_t(4) \cdot S_d + F_t(5) \cdot S_d + F_c(1) \cdot S_d - F_c(2) \cdot S_d - 10.0 \cdot \lambda = 0$$  

   (4)

5. Shear forces in the web panel zone: this zone is treated as a member loaded with internal and external loads, see figure 3. We represent the compressive forces in the joint by equally distributed loads (the length of the distribution is taken as $l_{\text{eff}}$, the spread area of the compressive force) and the tensile forces in the bolt-rows by concentrated loads. For each section in the member, we can express the shear force as function from the external force. For example, the following formula shows the shear force in section A-A:

$$V_{S_d} = 30.0 - F_t(3) \cdot S_d - F_t(4) \cdot S_d - F_t(5) \cdot S_d + F_c(2) \cdot S_d + F_t(8) \cdot S_d - F_c(4) \cdot S_d$$  

   (5)

5. PLASTIC CAPACITIES OF COMPONENTS

In joints with bolted end plate connections, the internal forces should be smaller than the following plastic capacities (ZOETEMEIJER, 1985; JASPART, 1991; EUROCODE 3, 1992).

1. If the column web is un-stiffened, the force acting in the point of compression should not exceed the compressive capacity of the column web, for example:

$$F_c(1) \cdot S_d \leq F_c(1) \cdot R_d$$  

   (6)

2. If the column web has no diagonal stiffeners, the force acting in the web panel should not exceed the shear capacity of the column web, for example:

$$|V_{S_d}| \leq V_{R_d}$$  

   (7)
3. The force in an individual bolt-row should not exceed the tension capacity of the end plate including the bolt-row, as if there are no other bolt-rows in the connection, see figure 4(a) and (b), for example:

\[ F_{t(2)} S_{d} \leq F_{t(2)} R_{d,pl} \]  

(8)

4. The forces in two or more bolt-rows should not exceed the tension capacity the end plate including these bolts-rows, see figure 4(c), for example:

\[ F_{t(2)} S_{d} + F_{t(3)} S_{d} + F_{t(1)} R_{d,pl} \]  

(9)

5. The forces in one or more adjacent bolt-rows should not exceed the tension capacity of these bolt-rows in the column flange as if there are no other bolt-rows.

6. The forces in one or more adjacent bolt rows should not exceed the tension capacity of the column web and the beam web as if there are no other bolt-rows.

7. The force acting in the point of compression should not exceed the capacity of the beam flange in compression.

6. SIMILARITIES WITH FRAME ANALYSIS

All characteristics appear to be linear and therefore similar to those of the problem of rigid plastic frame analysis:

<table>
<thead>
<tr>
<th>There is an objective</th>
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<tr>
<td>The external load acting at the frame should be maximised</td>
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</table>

with the following constraints:

- There should be equilibrium in the frame;
- Internal forces should not exceed plastic capacities of members.

Rigid plastic frame analysis can be carried out with the help of Linear Programming (LP) (MUNRO, 1977). Since these characteristics are similar, also cruciform joints can be analysed with linear programming.

7. TENSION BOLTS ACTING IN THE COMPRESSIVE ZONE

The LHS connection of the joint (see in figure 2) has an upper zone in tension and a lower zone in compression. It is obvious that \( F_{c(3),Sd} = 0 \) in the tension zone, and \( F_{t(8),Sd} = 0 \) in the compression zone. Sometimes it is less obvious to label a zone as “tension” or “compression”. The upper zone of the RHS connection, see figure 2, shows this. The moment acting in the RHS beam forces this zone in compression, but the normal force in the RHS beam works opposite. A requirement is that this zone is either compression \( (F_{t(1),Sd} = F_{t(2),Sd} = 0) \) or tension \( (F_{c(1),Sd} = 0) \). We can rewrite this requirement to:

\[ F_{t(1),Sd} + F_{t(2),Sd} \leq 0 \text{ or } F_{c(1),Sd} \leq 0 \]  

(10)

This is equivalent to (HILLIER & LIEBERMAN, 1967):

\[ F_{t(1),Sd} + F_{t(2),Sd} \cdot y M \leq 0 \]
\[ F_{c(1)} Sd + y M \leq M \]  

where \( M = \) should be an "extremely" large number  
\[ y = 0 \text{ or } 1 \]

M is an "extremely" large number, if M is equal to the maximum of:

- the sum of all capacities of the individual bolt rows in the end plate or column flange when loaded in tension and
- the capacity of the zone when loaded in compression.

In this example, we have to carry out two LP optimisations: for \( y = 0 \) and \( y = 1 \).

In a cruciform joint, we have 4 zones for this potential problem. Fortunately, with knowledge about the external forces, we can reduce the number of required optimisations. For example, we take the joint of figure 2.

- Due to \( M_l \), the upper zone in the LHS connection will act in tension and the lower zone in compression.
- With \( M_r \) in combination with \( N_r \) (tension), the lower zone in the RHS connection will act in tension.
- In the upper zone of the RHS connection \( M_r = 40 \text{ kNm} \) and \( N_r = 10 \text{ kN} \) acting 0.19 meters down the upper beam flange have the opposite effect. We can replace this moment and normal force by \( N'_r = 10 \text{ kN} \) acting 4.19 meters down the upper beam flange. We can conclude that the upper zone of the RHS connection will act in compression, because of the location of \( N'_r \). Only if \( N'_r \) acts in the lower part of the connection, the zone can be either tension or compression.

The given method can be extended to the general case. In a cruciform joint the maximum number of LP optimisations is 4. In most practical cases one optimisation will be sufficient.

### 8. OPTIMAL FORCE DISTRIBUTION FOR SHEAR VERIFICATION

An LP-problem can have so-called multiple optimal solutions, i.e. more than one solution (HILLIER & LIEBERMAN, 1967). Although the optimum will be equal, the solutions themselves can be very different. Multiple optimal solutions also occur when we determine the force distributions in a joint. Figure 5 illustrates this. The joint in this figure is subjected to \( M = 40 \text{ kNm} \). There should be equilibrium in the joint. We assume that all plastic capacities are satisfied except those the bolt-rows in the column flange. These are restricted to 70 kN. The right part of the figure shows two feasible force distributions in the joint that satisfy plasticity. The difference between the two force distributions is that the sum of all the bolt-row forces in the left solution (82.5 kN) is lower than the sum of the bolt-row forces in the right solution (94 kN). So, shear in the beam (\( V = 35 \text{ kN} \)) will verify more easily with the left force distribution than the right one. In the left case, the bolts are less loaded in tension and can be used for shear.

A general way to ensure an optimal force distribution for the check of shear resistance is to minimise the sum of all bolt-row forces \( F_{c(i)} Sd \) (which is defined as function \( Z \)) after the optimum \( \lambda = \lambda_{max} \) is reached. Simultaneously, a constraint should be added to the problem: \( \lambda = \lambda_{max} \). This constraint
ensures to keep $\lambda = \lambda_{\text{max}}$. The forces in the bolt-rows will be distributed as far as possible from the points of compression. This is compatible with Eurocode 3.

9. EXAMPLE

Figure 6 shows the matrices with constraints for the joint given in figure 2. For the purpose of this paper, the size of the matrices is reduced by leaving $F_t(8).S_d$, $F_c(2).S_d$ and $F_c(3).S_d$ out of the problem (these variables will be 0 anyway). The only capacities considered are failure of the column flange with bolts, yielding of the column web in compression and shear of the column web in section A-A.

The optimisations to be carried out are: maximise $z_1 = \lambda$. After the optimum is reached, $z_2 = Z$ has to be maximised under the additional constraint that $\lambda = \lambda_{\text{max}}$. Rows 1 to 4 in figure 6 show the equilibrium of the LHS and RHS connections (equations (1) to (4)). Rows 5 to 16 show plasticity constraints. Rows 17 and 18 represent the shear constraints in section A-A of the column web panel (equations (5) and (7) in combination). Row 19 says that $Z$ equals to the sum of all bolt row forces. Row 20 restricts the solution to $\lambda \leq 1$. Rows 21 and 22 represent the problem described in equation (11).

The solution of the problem is: $F_t(1).S_d = 0; F_t(2).S_d = 0; F_t(3).S_d = 0; F_t(4).S_d = 25.517; F_t(5).S_d = 80.0; F_c(1).S_d = 95.517; F_t(6).S_d = 51.429; F_t(7).S_d = 0; F_c(4).S_d = 51.429; \lambda = 1; Z = 156,946.$

10. IMPLEMENTATION

The here presented method has been implemented in CASTA/Connections version 2.10. This program is developed at TNO. It determines the mechanical properties of joints with bolted end plate connections. CASTA/Connections is widely used by practitioners in the Netherlands. The characteristics of this implementation are:

- a reduction of the amount of program-code compared with the old version of the program;
- the program now contains a general procedure for the determination of force distributions in joints;
- the number of iterations necessary to find the optimal solution for a practical joint is less than 10. In other words, the algorithm is numerical stable.
- the time required to carry out the optimisation is a fraction of a second (MS-DOS 486 machine)
11. CONCLUSIONS

Eurocode 3 contains a procedure for the determination of the force distribution in beam-to-column moment joints based on plasticity. This procedure is aimed at both hand and computer calculations. For computer applications, however, it seems more feasible to adopt a plastic approach in combination with linear optimisation. This alternative is general and allows for all possible joints and in-plane loading cases.

KEYWORDS

Steel Structures, Joints, Beam-to-Column Connections, Bolted Connections, End Plate Connections, Eurocode, Linear Programming.

REFERENCES

figure 1: End plate connection in a beam-to-column joint.

figure 2: Extended end plate connection, cruciform joint.
figure 3: Shear forces in the column web panel.

(a) Individual capacity row (2)  
(b) Individual capacity row (3)  
(c) Capacity of row (2) + (3)

figure 4: Yield lines for individual bolt-rows and groups of bolt-rows in the end plate.

figure 5: Feasible force distributions in one joint.
figure 6: Constraints for the joint given in figure 2.