Strategies for Economic Design of Unbraced Steel Frames

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ABSTRACT

Experience in different European countries demonstrates that the design process of unbraced frames is economically very delicate, because decisions made during predesign have a major impact on the fabrication costs. Especially when the costs of a steel structure are calculated on a kilogram basis, a designer may decide to optimise the frame for minimum weight. In general, minimum weight solutions are obtained when the joints in the frame are rigid. Rigid joints usually have to be stiffened and therefore lead to high fabrication costs. Studies have shown that minimum weight solutions may be up to 20% more expensive than solutions where also the fabrication costs have been taken into consideration to optimise the design.

If a designer focuses on optimal economy during frame design, the design process is less simple, compared to a frame design for minimum weight. Beam and column properties as well as the properties of the joints should be selected in such a way that economy can be achieved during fabrication.

To help practitioners to find the most economic design of unbraced frames, this paper gives design strategies. It focuses on elastic frame design, since in practice this is used most. Tables are given to support the various design steps.

KEYWORDS

Predesign, Joints, Connections, Unbraced Frames, Economy, Semi-rigid, Rigid.

LIST OF SYMBOLS

\(a\) throat thickness of a fillet weld;

\(f\) cost ratio between 1 kg steel including labour and anti-corrosion protection and 1 kg welding material including labour;

\(k_x\) stiffness factor dependent on the type of joint relative to the lever arm;

\(l_b\) beam span;

\(l_c\) storey height;

\(t_{fc}\) column flange thickness;

\(t_{wc}\) column web thickness;

\(z\) distance between centre of compression and tension;

\(E\) Young’s modules;

\(F\) horizontal load acting on a storey;

\(F_t\) added horizontal load;

\(I_b\) moment of inertia of the beam;

\(I_{br}\) moment of inertia of the beam in a design with rigid joints;

\(I_{bs}\) moment of inertia of the beam in a design with semi-rigid joints;

\(I_c\) moment of inertia of the column;

\(I_{cr}\) moment of inertia of the column in a design with rigid joints;

\(I_{cs}\) moment of inertia of the column in a design with semi-rigid joints;

\(S_j\) initial stiffness \(S_{j,ini}\);

\(S_{j,act}\) initial stiffness \(S_{j,ini}\) calculated according to Eurocode 3 or another design standard;

\(S_{j,app}\) 'good guess' of the initial stiffness \(S_{j,ini}\);

\(\rho\) specific gravity of steel, \(\rho = 7850 \text{ kg} / \text{m}^3\)

INTRODUCTION

The economy in steel frames is depends largely upon the final detailing of the joints. Bjorhovde & Colson (1992) report on cost differences between design alternatives of 20% on the bare steel frame. In practice, however, the freedom to investigate alternatives is often limited when it comes to the design of the joint, because in practice the design of a steel frame is normally a two phase procedure, see Figure 1. In the first phase the frame is designed. This means that beam and column sections are chosen and that the frame is checked for stability, strength and serviceability requirements. Also a conceptual decision is taken about the joints to be applied. In unbraced frames, normally rigid joint are assumed while elastic frame analysis is adopted. In many cases the price is fixed based on the weight of beam and column sections. If rigid joints are assumed in unbraced frames, the weight of the structure is minimal, thus the price is low. Indeed, this seems to result in ‘optimal’ solutions.

Problems arise in the second phase of the design process, when the joints have to be designed. Since it was assumed in the first phase that the joints are rigid, they should be designed to fulfil a minimum rigidity criterion. Modern codes, like Eurocode 3 (1994) include such a criterion. In normal circumstances, this criterion determines the lay-out of the joints. As a consequence, due to the fact that the joint should behave sufficiently rigid, stiffeners are required. The “optimal” light weight solution suddenly becomes very expensive (Weynand et al, 1998).

If in the second phase appears that the joints require stiffening, another option could be to go back to the first design phase and change the beam and column sizes. This may lead to unstiffened (semi-rigid) joints and lower added costs of both materials and fabrication. In practice, however, this change of column and beam sizes will only occur in rare cases. There are two reasons for this:
Figure 1: Design process of a steel frame with rigid joints and elastic frame analysis (after ECSC, 1997).

- the first and second phase of the design process are not always carried out by the same party. In many cases, the design of beam and columns is carried out by the engineering office whilst the design of the joints is carried out by the steel fabricator. This hampers the communication necessary to adopt more economical detailing of joints.
- quite often beam and column section have been ordered immediately after finalising the first design phase, so by the time the joints are designed, there is no room for changing section properties.

To avoid the difficulties described above, the economy of a frame including the joint detailing can be judged best in the conceptual stage of the design process. In that case, the designing engineer needs to have design tools available, allowing him to choose between rigid and semi-rigid solutions for joints. In this paper, some of these tools will be given.
The application of these tools is restricted to European H and I sections. Some further restrictions apply to end
plated joints:

a) the connection has two bolt rows in tension;
b) the bolt diameter is approximately 1.5 times the thickness of the column flange;
c) the location of the bolt is close to the root radius of the column flange, the beam flange and the web;
d) the end plate thickness is similar to the column flange thickness.

This type of end plate joints is frequently applied in practice and these restrictions usually lead to good efficiency in
structural and economical respect.

PREDESIGN WITH RIGID JOINTS

Normally, when designing unbraced frames, there is one design criterion which determines the beam and column
sizes, i.e., the limitation of the horizontal deflections of the frame. It is advisable for an engineer to determine the
beam and column sizes during predesign based on a check of horizontal deflections of the frame. It requires only a
simple frame analysis and a check to a criterion like “the overall horizontal deflection of the frame in serviceability
limit state should be less than the overall height of the frame divided by X”. The value of X is dependent on the
country where the frame will be built When this criterion is satisfied, normally also other criteria, like section
strength, column and beam stability, overall frame stability, etc. will be satisfied.

In practice, the frame is usually assumed to behave continuously in the joints. This means, that the joints should be
sufficiently rigid. This assumption leads to a simple model in the global frame analysis, because stiffness properties
of the joints does not need to be considered in this analyses. The assumption of rigid joints is advantageous for the
engineer designing the frame. With a minimum of design effort, he is able to determine the section properties of the
members of the frame. However, to be sure about the economy during the final design of the joints, it is important
for the engineer to predesign the joints.

When rigid joints are assumed, the joints should be sufficiently rigid to justify the design assumption made in global
frame analysis, because otherwise the horizontal deflections are more or the stability is less than assumed.
Eurocode 3 (1994) gives a simple criterion to classify joints as rigid:

\[ S_j > \frac{25 E I_b}{l_b} \]  

(1)

The determination of the joint stiffness \( S_j \) requires knowledge about the lay-out of the joint. In the conceptual
design stage of the process this lay-out is difficult to assess. Fortunately, a design table, see Table 1, exists to make
a good guess of the joint stiffness without going too far in detail.

Ryan (ECSC, 1997) used Table 1 in combination with the Eurocode 3 criterion as illustrated with the following
example. Assume an HE 340 B column connected to an IPE 500 beam. The beam span is 7.2 m. The joint is
assumed to behave rigid. An extended end plate without column web stiffeners is assumed.

If the joint should behave rigid, then based on formula (2) and Table 1:

\[ S_j \approx \frac{E \cdot z^2 \cdot t_{lc}}{k_x} > \frac{25 E I_b}{l_b} \]  

hence

\[ z > \sqrt{\frac{25 k_x I_b}{t_{lc} l_b}} = \sqrt{\frac{25 \cdot 13 \cdot 67120 \cdot 10^4}{21.5 \cdot 7200}} = 1200 \text{ mm} \]  

(4)
Table 1: Good guess formulae to assess the stiffness of a joint (taken from Steenhuis et. al., 1994)

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$S_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extended end-plate and unstiffened</td>
<td>$\frac{E z^2 t_{l,w}}{13}$</td>
</tr>
<tr>
<td>Extended end-plate, stiffened in tension and compression</td>
<td>$\frac{E z^2 t_{l,w}}{8.5}$</td>
</tr>
<tr>
<td>Extended end-plate and Morris stiffener</td>
<td>$\frac{E z^2 t_{l,w}}{3}$</td>
</tr>
<tr>
<td>Flush end-plate</td>
<td>$\frac{E z^2 t_{l,w}}{14}$</td>
</tr>
<tr>
<td>Flush end-plate and cover plate</td>
<td>$\frac{E z^2 t_{l,w}}{11.5}$</td>
</tr>
<tr>
<td>Welded joint and unstiffened</td>
<td>$\frac{E z^2 t_{l,w}}{11.5}$</td>
</tr>
<tr>
<td>Welded joint stiffened in tension and compression</td>
<td>$\frac{E z^2 t_{l,w}}{5.5}$</td>
</tr>
</tbody>
</table>

In other words, if $z \approx 1200$ mm then the Eurocode 3 (1994) criterion is just satisfied. This does not take into account uncertainties in the “good guess” stiffness from Table 1. Therefore, it is assumed that the lever arm $z$ about 1300 mm.

Since the overall depth of the beam is 550 mm, a haunch of 750 mm height needs to be adopted, see Figure 2. This is very expensive!
So, it can be concluded that the design rules of Eurocode 3 and Table 1 give a simple guidance for a structural engineer to determine the type of rigid joint during the conceptual frame design. If an engineer finds that the application of rigid joints leads to expensive stiffeners, he might wish to investigate the possibilities of semi-rigid joints. In the next section, some guidance will be given for this step.

INVESTIGATION OF ALTERNATIVES TO RIGID JOINTS

When a designer selected beam and column sections with the assumption of rigid joints, it is not possible to adopt semi-rigid joints without increasing the beam or column sections, because in an economical design, the beam and column dimensions are chosen in such a way, that the governing design criteria (normally frame deflections) are just met. The application of semi-rigid joints without increasing beam or column dimensions simply means that these limits are exceeded.

Table 2: The effect of the increase of beam of column dimension on required stiffness of the joint

<table>
<thead>
<tr>
<th>Joint type</th>
<th>Only beam increased</th>
<th>Only column increased</th>
<th>Both beam and column increased</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cruciform joint or knee joint</td>
<td>( \frac{1}{S_j} \lt \frac{l_s}{6E} \left( \frac{1}{I_{c,r}} - \frac{1}{I_{c,s}} \right) )</td>
<td>( \frac{1}{S_j} \lt \frac{l_s}{6E} \left( \frac{1}{I_{b,r}} - \frac{1}{I_{b,s}} \right) + \frac{l_s}{6E} \left( \frac{1}{I_{c,r}} - \frac{1}{I_{c,s}} \right) )</td>
<td>( \frac{1}{S_j} \lt \frac{l_s}{6E} \left( \frac{1}{I_{b,r}} - \frac{1}{I_{b,s}} \right) + \frac{l_s}{6E} \left( \frac{1}{I_{c,r}} - \frac{1}{I_{c,s}} \right) + \frac{l_c}{12E} \left( \frac{1}{I_{c,r}} - \frac{1}{I_{c,s}} \right) )</td>
</tr>
<tr>
<td>T joint with beams on either side</td>
<td>( \frac{1}{S_j} \lt \frac{l_s}{6E} \left( \frac{1}{I_{c,r}} - \frac{1}{I_{c,s}} \right) )</td>
<td>( \frac{1}{S_j} \lt \frac{l_s}{3E} \left( \frac{1}{I_{c,r}} - \frac{1}{I_{c,s}} \right) )</td>
<td>( \frac{1}{S_j} \lt \frac{l_s}{3E} \left( \frac{1}{I_{c,r}} - \frac{1}{I_{c,s}} \right) + \frac{l_c}{12E} \left( \frac{1}{I_{c,r}} - \frac{1}{I_{c,s}} \right) )</td>
</tr>
<tr>
<td>T joint with continuous column</td>
<td>( \frac{1}{S_j} \lt \frac{l_s}{12E} \left( \frac{1}{I_{c,r}} - \frac{1}{I_{c,s}} \right) )</td>
<td>( \frac{1}{S_j} \lt \frac{l_s}{12E} \left( \frac{1}{I_{c,r}} - \frac{1}{I_{c,s}} \right) )</td>
<td>( \frac{1}{S_j} \lt \frac{l_s}{12E} \left( \frac{1}{I_{c,r}} - \frac{1}{I_{c,s}} \right) + \frac{l_c}{12E} \left( \frac{1}{I_{c,r}} - \frac{1}{I_{c,s}} \right) )</td>
</tr>
</tbody>
</table>
Now assume that a designer determined beam and column properties with the assumption of rigid joints, but he wants to investigate the effect of enlarging either beam or column dimensions on the joint detailing. For this investigation, Table 2 can be used. The background of the formulae in this table can be explained as follows.

Assume an unbraced multi-storey, multi-bay frame as shown in Figure 3.

![Unbraced frame and a sub-system for a cruciform joint configuration.](image)

Figure 3: Unbraced frame and a sub-system for a cruciform joint configuration.

In this frame, a sub-system is considered (see right side of Figure 3). The horizontal deflection of this sub-system can be calculated as follows

\[ \delta = \frac{F_t l_b l_c^2}{12 E I_b} + \frac{F_t l_c^2}{2 S_j} + \frac{F_t l_c^3}{12 E I_c} \]  

(5)

For simplicity, the horizontal load \( F \) acting at the floor level of the sub frame is neglected.

Now assume that two different frame designs are being compared with each other. Of both designs it is required that they have the same horizontal deflection. One frame is a frame with rigid joints, hence \( S_j \) is infinite, \( I_b = I_{b,r} \) and \( I_c = I_{c,r} \). The other frame is a frame with semi-rigid joints, hence \( S_j \) is finite, \( I_b = I_{b,s} \) and \( I_c = I_{c,s} \)

\[ \delta_{\text{rigid frame}} = \delta_{\text{semi-rigid frame}} \]  

(6)

\[ \frac{F_t l_b l_c^2}{12 E I_{b,r}} + \frac{F_t l_c^3}{12 E I_{c,r}} = \frac{F_t l_b l_c^2}{12 E I_{b,s}} + \frac{F_t l_c^2}{2 S_j} + \frac{F_t l_c^3}{12 E I_{c,s}} \]  

(7)

thus:
\[
\frac{1}{S_j} = \frac{l_b}{6 \, E \left( \frac{1}{I_{b,r}} - \frac{1}{I_{b,s}} \right)} \cdot \frac{l_c}{6 \, E \left( \frac{1}{I_{c,r}} - \frac{1}{I_{c,s}} \right)}
\]  

This is reported in Table 2. The other equations in the table can be derived on a similar basis.

In Table 2 the symbols \( l_c \) and \( l_b \) are used. It is quite straightforward to determine these properties in case of regular spacing of storeys and bays. However in some cases some additional guidance is required. For example, with different beam spans on either side of a cruciform or a horizontal T-joint take for \( l_b \) the average of both spans. If the storey heights on either side of a cruciform or a vertical T-joint are different, take for \( l_c \) the average of both storey heights.

**EXAMPLE**

In the previous example on the predesign of a rigid joint, the engineer wants to investigate the effect of the increase of the column from an HE 340 B to an HE 400 B. The storey height is 3.5 m. The required joint stiffness can be determined with Table 2:

\[
\frac{1}{S_j} < \frac{l_c}{12 \, E \left( \frac{1}{I_{c,r}} - \frac{1}{I_{c,s}} \right)} = \frac{3500}{12 \cdot \frac{36660}{210000} \cdot 10^4 \cdot \frac{1}{57680 \cdot 10^4}} = 1.38 \cdot 10^{-12} \text{ rad} / \text{Nmm} \quad (9)
\]

Hence

\[
S_j > 724 \cdot 10^9 \text{ Nmm/rad} \quad (10)
\]

A joint with a stiffness of \( S_j = 724 \cdot 10^9 \text{ Nmm/rad} \) would lead to a haunch of 900 mm surprisingly. Despite the increase in column size this is more than required for the rigid joint of Figure 2. The reason is that a rigid joint in reality is not infinite rigid (\( S_j = \infty \)), but designed with the Eurocode 3 stiffness classification criterion for rigid joints:

\[
S_j > \frac{25 \, E \, I_b}{l_b} \approx \frac{25 \cdot 210000 \cdot 67120 \cdot 10^4}{7200} \approx 490 \cdot 10^9 \text{ Nmm/rad} \quad (11)
\]

By using this criterion, the bearing capacity of the frame will not drop more than 5\% compared to a frame with infinitive rigid joints. In other words, the rigid joint (\( S_j = 490 \cdot 10^9 \text{ Nmm/rad} \)) has a lower stiffness than the stiffness used in the frame analysis (\( S_j = \infty \)). Assume that \( S_j = 724 \cdot 10^9 \text{ Nmm/rad} \) is used in the frame analysis, what minimum stiffness should be used to avoid more than 5\% effect on the bearing resistance of the frame? In Steenhuis et al (1994) some formulae are given for semi rigid joints to check the required accuracy of joint stiffness in a design. These formulae are given in Table 3:

**Table 3: Criteria for accuracy of joint stiffness**

<table>
<thead>
<tr>
<th>Approximation of joint stiffness used in the frame analysis</th>
<th>Requirements for the ‘actual’ joint stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_{j,\text{app}} &lt; \frac{24 , E , I_b}{l_b} )</td>
<td>( \frac{24 , S_{j,\text{app}} , E , I_b}{30 , E , I_b + S_{j,\text{app}} , l_b} \leq S_{j,\text{act}} \leq \frac{30 , S_{j,\text{app}} , E , I_b}{24 , E , I_b - S_{j,\text{app}} , l_b} )</td>
</tr>
<tr>
<td>( S_{j,\text{app}} \geq \frac{24 , E , I_b}{l_b} )</td>
<td>( \frac{24 , S_{j,\text{app}} , E , I_b}{30 , E , I_b + S_{j,\text{app}} , l_b} \leq S_{j,\text{act}} \leq \infty )</td>
</tr>
</tbody>
</table>

If these requirements are fulfilled, the bearing capacity of a frame with a stiffness \( S_{j,\text{act}} \) will differ less than 5\% from the same frame with joint stiffness \( S_{j,\text{app}} \).
The lower requirement of Table 3 can be rewritten as:

\[
S_{j,act} \geq \frac{24 \cdot S_{j,app} \cdot E \cdot I_b}{30 \cdot E \cdot I_b + S_{j,app} \cdot h_b}
\]  

(12)

With aid of this formula, a lower bound value for the stiffness can be assessed. Assume \( S_j = S_{j,app} = 724 \cdot 10^9 \) Nmm/rad is adopted in the frame analysis.

Hence:

\[
S_{j,act} \geq \frac{24 \cdot 724 \cdot 10^9 \cdot 210000 \cdot 67210 \cdot 10^4}{30 \cdot 210000 \cdot 67210 \cdot 10^4 + 724 \cdot 10^9 \cdot 7200} = 260 \cdot 10^9 \text{Nmm/rad}
\]  

(13)

This is considerably lower than \( S_j = 724 \cdot 10^9 \) Nmm/rad.

The pre-design can now be carried out with the help of Table 1:

\[
S_j \approx \frac{E \cdot z^2}{k_x} \cdot t_{fc} \cdot k_x
\]  

(14)

\[
z > \sqrt[2]{\frac{S_j \cdot k_x \cdot t_{fc} \cdot E}{24 \cdot 210000}} = 810 \text{ mm}
\]  

(15)

If \( z = 810 \) mm, it is really a minimum value. Since the good guess formula (Error! Not a valid link.) has some inherent inaccuracy, it is better to adopt a slightly higher value. It is assumed that \( z = 900 \) mm. This means that the haunch needs to have a height of \( 900 - 550 = 350 \) mm. The haunch height is considerably reduced from 750 mm to 350 mm.

![Figure 4: A semi-rigid joint between an HE 400 B column and an IPE 550, beam span 7.2 m](image)

A rough assessment of the cost comparison of these two alternatives is given in Table 4. The haunch is to be assumed built from plates and welded around with 12 mm fillet welds (throat thickness). The length of the haunch is to be assumed 1.5 time its height. This yields in a weld volume of \( 2 \cdot 4,3 \cdot h_b \). The factor \( f \) is the cost ratio between 1 kg welding material including labour and anti corrosion protection and 1 kg steel including labour. This
ratio is dependent on country, automation grade of the company and labour costs. Here it is assumed as 100 (van Douwen, 1979). It should be noted that the decrease of plate material of the haunch is neglected.

**Table 4: Cost comparison**

<table>
<thead>
<tr>
<th>Cost category</th>
<th>Rigid joint</th>
<th>Semi-rigid joint</th>
<th>Savings when semi-rigid joints are used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column material (kg steel) A, l, ρ</td>
<td>171 · 10^4 3,5 · 7850 = 470</td>
<td>198 · 10^4 · 3,5 · 7850 = 544</td>
<td>- 74 kg</td>
</tr>
<tr>
<td>Weld material haunch (kg steel) a² · 4,3 · hₗ · ρ · f</td>
<td>0.012² · 4,3 · 0,75 · 7850 · 100 = 364 kg</td>
<td>0.012² · 4,3 · 0,35 · 7850 · 100 = 170 kg</td>
<td>+ 194 kg</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>+ 120 kg</td>
</tr>
</tbody>
</table>

In conclusion, it can be stated that despite the use of more column material, it is beneficial to adopt semi-rigid joints because of savings in welding cost.

**FINAL DESIGN OF THE FRAME AND THE JOINTS**

If a predesign of the frame with semi-rigid joints is made, the joint stiffness as found with the help of Table 2 should be introduced in the frame analysis and further code checks.

In the second phase of the design process, the joints should be designed. The ‘actual’ stiffness of the joint should be sufficiently close to the stiffness found with Table 2. For this verification, the formulae in Table 3 of the previous paragraph should be used.

**CONCLUSIONS**

The paper shows how a structural engineer can get an impression of the possible lay-out of rigid joints in a very early design stage of a design process. The method is simple and straightforward. A limited amount of design information is required.

If the designer is dissatisfied with the design of the rigid joints, he can investigate the effect of the increase of beam or column sections on the lay-out of the joints. The joints normally will behave semi-rigid. The lay-out can be checked with a simple hand method, based on the principle that the solution with semi-rigid joints should have the same performance as the solution with rigid joints.

Finally, examples demonstrate the feasibility of the proposed method.

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