Halley’s method of durations
Its history and appliance

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Introduction
The approaching transit of Venus on 2004 June 8 has generated unprecedented attention from a general public. Since it will be visible from start to end from almost all of Europe, Africa and the western parts of Asia, this particular transit is generally regarded as the most favourable one of the present century’s pair to witness Venus’ black silhouette traversing the brilliant disc of our sun. Several international projects have already been launched, offering an excellent opportunity for both novice and experienced amateur astronomers to take part in the observation of this rare celestial event throughout the world.

Among other things, these projects aim at ascertaining the sun’s mean parallax, and hence its mean distance from the earth, by observing the transit of Venus, just like astronomers of the past had this determination as the main object of their many expeditions. Though the sun’s distance was established very accurately in recent years by means of radar measurements and the forthcoming transit of Venus therefore serves no longer any scientific purposes, still, finding the sun’s distance is rightly regarded to be of challenging educational interest.

In order to arrive at the sun’s distance from observations of the transit of Venus, the appliance of the so-called method of durations — sometimes referred to as Halley’s method — is common to most projects. It implies the comparison of the transit’s durations as measured from two different and preferably widely separated points on the earth. In this article I will expound the method’s main principles and discuss the early history of its development.

The genesis of a new idea
There is no indication whatever that the observations of the first couple of seventeenth century transits were used to establish any other solar system dimensions, but to make an estimate of the apparent angular sizes of Mercury and Venus. Truly, Jeremiah Horrocks gave a conjecture about the value of the sun’s parallax in his *Venus in sole visa*, but it was based on mere geometrical aesthetics rather than concluded from the transit’s duration. In the last pages of his account, he held the prospect of a separate treatise in which he would give many other certain and easy methods of providing the distance and magnitude of the sun, but to say that this would support the idea of Horrocks being supposingly the first to notice the connection between the solar parallax and the duration of a transit would be rather speculative.

In 1663 things became more explicit when Scottish scientist James Gregory (1638–1675) published his *Optica Promota*, a book on optics, with an appendix containing several astronomical problems and their solutions. One of these problems dealt with the determination of a planet’s parallax by measuring the duration of its conjunction with another heavenly body. Though the solution to this problem was set out in general terms, the method could, according to Gregory, be advantageously applied to the particular case of the inner planets Mercury and Venus transiting the sun’s disc during their inferior conjunctions:

> This problem has an excellent, but nevertheless laborious use in observing Venus or Mercury partially obscuring the Sun, as from this, the solar parallax may be found. [1]

Indeed, the solution Gregory presented was tedious, involving complex spherical trigonometry. Nor did he elucidate how his proposed method of durations should be applied to the transits of Venus or Mercury to establish the sun's parallax. Nonetheless, Gregory is to be esteemed for his keen observation of the employment of the transits of Venus and Mercury, planetary phenomena of which existence man had knowledge then for just over thirty years.

Presumably, young undergraduate Edmond Halley (1656–1742) knew of Gregory's suggestion when he was to observe the transit of Mercury across the sun in the island of St Helena on 1677 November 7. Halley later recalled that, upon noticing that the time between the two interior contacts could be found without an error of a single second, he immediately concluded that the sun's parallax might be duly determined by such observations. As he wrote to his patron Jonas Moore on December 2:

> I have notwithstanding had the opportunity of observing the ingress and egress of [Mercury] on the [Sun], which compared with the like Observations made in *England*, will give a demonstration of the Suns Parallax, which hitherto was never proved, but by probable arguments. [2]

On 1678 February 24, Moore reported to the Royal Society that Halley had successfully observed the transit of Mercury from St Helena. The next week, another observation by Jean Charles Gallet at Avignon was spoken of and in this connection Halley's letter was mentioned again by Robert Hooke, pointing out that the parallax and distance both of the sun and Mercury might be experimentally verified from the two observations. When Halley had returned from St Helena, he made a conjecture about the solar parallax using his single observation of the transit's duration, assuming the least distance of Mercury from the sun's centre according to Gallet's observations and taking the movement of Mercury from the astronomical tables of Thomas Streete. Without giving hints for the precise procedures of his calculation, Halley, in an appendix to his 1679 *Catalogus Stellarum Australium*, thus attributed 45 seconds of arc to the solar parallax, a value he knew to be very rough and rather fanciful. For he was well aware of the more plausible results obtained by the determination of the diurnal parallax of Mars by the direct measurement of the planet's angular distance from the fixed stars, but this particular method demanded impracticable care:

> For what observer, or what instrument may ascertain the celestial angles without exceeding an error of five seconds? If someone would venture to pride himself on doing this, I should believe that he will expose himself to pass for being completely ignorant and without experience with regard to this matter. [3]

There remained the observation of the next transit of Venus to provide a prosporous opportunity to establish the sun's distance from the earth with a greater certainty than would be possible otherwise:

> For then, the parallax of Venus from the sun may be found by the method I just explained, and all required observations will be facilitated by [this parallax] being three times as much as that of the sun, so that from this phenomenon any mortal may gain an understanding of this knowledge. [4]

Many years later Halley finally presented his method as a programme of work which would ultimately emanate an enormous influence. Probably occasioned by the last transit of Mercury on 1690 November 10, he first announced his method of durations in a dissertation on the visibility of the planets' inferior conjunctions with the sun, which was read to the Royal Society on 1691 October 3. Next to an in depth discussion of various elements and circumstances of future transits of Venus, he almost casually mentioned their beneficial use:

> This sight, which is by far the noblest astronomy affords, like the secular games, is denied to mortals for a whole century, by the strict laws of motion. It will be afterwards shown, that by this observation alone, the distance of the sun, from the earth, might be determined with the greatest certainty, which, on account of the parallax otherwise quite insensible, has not hitherto been precisely defined. [5]

As the transit of Mercury wasn't mentioned at all in this respect, it is likely that Halley already bore in mind that the determination of the sun's distance by the transit of Venus would not be equally applicable to the transit of Mercury. In order to stress that the concluded solar parallax would gain accuracy, Halley strongly contrasted his methodology and the usual trigonometric measurements:

> The principal use of these conjunctions is accurately to determine the distance of the sun from the earth, or his parallax, which astronomers have by several methods attempted in vain, while the smallness of the angles sought do easily elude the nicest instruments. [6]

Most certainly, Halley referred once again to the unsatisfactory observations by Cassini and Flamsteed of the diurnal parallax of Mars during its oppositions in 1672 and 1679. As a matter of fact, Halley had taken an active part in Flamsteed's observations on the latter occasion and probably called on his practical experience when he wrote down these words. He realized that another approach was needed, which was conveniently provided by the apparent movement of Venus across the sun's disc of about four minutes of arc per hour, allowing angles to be expressed in units of time. Against a direct measurement of angles, Halley therefore set a measurement of time, which in effect would yield more accurate results:

> But in observing the ingress of Venus into, and egress from, the sun, the space of time between the moments of the internal contacts may be obtained to a second of time, that is, to 1/15 of a second, or 4" of the observed arch, by means of an ordinary telescope and clock that goes accurately for 6 to 8 hours. On the next occasion I will show that from just two of such observations made at places fit for this purpose, the sun's distance may be found with an accuracy of one part in five hundred. [7]

For the moment Halley left aside a full account of the rudiments of his proposal. Yet a detailed continuation of his dissertation, explicating the principles of his invented method of durations, wasn't published until 1716.

Halley's admonition

In his 1716 paper Halley presented an extensive account of his devised utilisation of the transit of Venus to determine the sun's distance from the earth. Notably, the memoir was written in Latin, indicating that Halley thought his paper was of international importance. He first outlined the general foundations of the method and then pointed out several proper places for observing the complete duration of the forthcoming transit. Although the next transit of Venus wasn't yet to be seen for over forty years, this paper didn't treat the matter as of merely theoretical interest. On the contrary, it contained a passionate call for action:

> And indeed I could wish that many observations of the same phenomenon might be taken by different persons at several places, both that we might arrive at a greater degree of certainty by their agreement, and also lest any single observer should be deprived, by the intervention of clouds, of a sight, which I know not whether any man living in this or the next age will ever see again; and on which depends the certain and adequate solution of a problem the most noble, and at any other time not to be attained to. I recommend it therefore, again and again, to those curious Astronomers, who (when I am dead) will have an opportunity of observing these

> things, that they would remember this my admonition, and diligently apply themselves with all their might to the making of this observation. [8]

Owing to the effects of parallax, arising from both the latitude of the place and its diurnal motion imparted by the earth's rotation, the ingress and egress as seen from a point on the surface of the earth would appear a little earlier or later than when viewed from the centre of the earth. As a result, the apparent stay of Venus on the sun's disc would vary according to an observer's location. Therefore, if the transit of Venus was seen from two distant places, the observed durations between the interior contacts would differ from each other by a sensible amount of time. The difference between the durations could theoretically be obtained by assuming a certain known quantity for the parallax of the sun. If this difference was found to be greater or less by observation, then, according to Halley,

> … the Sun's parallax will be greater or less, in the same proportion. [9]

So, the sun's assumed parallax was to its parallax found by observation as the calculated difference was to the observed difference. Although not explicitly stated by Halley, this fundamental reasoning was based on the sole supposition that the retarding and accelerating effects of parallax upon the times of interior contact were directly proportional to the solar parallax. Because of this understood proportionality, the ratio of the assumed solar parallax to the calculated difference wasn't subject to the choice of the starting value.

On the supposition that the sun's parallax was twelve and a half seconds of arc, Halley found that for the next transit of Venus the greatest difference to be expected was between locations in the East Indies, where Venus' stay was shortened by eleven minutes, and Hudson Bay, where it was lengthened by six minutes. As each arc second of parallax would consequently correspond to a difference of about 80 seconds, the quantity of the sun's parallax could well be obtained to within the fortieth part of one arc second — i.e. to within a five-hundredth part of what it really was — provided that the difference between the durations was determined to within two seconds.

Halley also explained why the transit of Venus, despite its rather infrequent recurrence, was a more favourable opportunity for determining the sun's distance than the transit of Mercury. As the parallax of Venus differed almost four times as much from that of the sun as the parallax of Mercury, the effect of errors in the determination of the difference between the durations on the value of the solar parallax would correspondingly be four times smaller in the case of Venus. Therefore, according to Halley, observations of the transits of Venus would be ideal for the determination of the sun's distance. These observations could already be secured without

> … any other instruments (…) than common telescopes and clocks, only good of their kind; and in the observers, nothing more is needful than fidelity, diligence, and a moderate skill in Astronomy. For there is no need that the latitude of the place should be scrupulously observed, nor that the hours themselves should be accurately determined with respect to the meridian: it is sufficient the clocks be regulated according to the motions of the heavens, if the times be well reckoned from the total ingress of Venus into the Sun's disc to the beginning of her egress from it; that is, when the dark globe of Venus first begins to touch the bright limb of the Sun. [10]

The theoretical principles and the practical implications now set out, there remained the issue of obtaining the difference between the durations theoretically, starting from an assumed value of the solar parallax. In the continuation of his discourse, Halley presented a graphical process to illustrate an approach of dealing with this problem, which was accompanied by a large figure and a minute explication.

The geometrical construction involved a perspective projection of the phenomenon, representing the sun's limb, the inclined path of Venus and its disc, and the earth's globe respectively. The distance of the plane of projection was the radius vector of Venus and the point of sight was the centre of the sun. The circle representing the earth's globe, as projected onto the plane of the picture, was therefore placed in the centre and measured the difference between the horizontal parallaxes of Venus and the sun, this being the maximum parallax of an observer on the earth's surface. Calibrated by the instant of Venus' least distance from the sun's centre, an absolute time scale was fitted along the straight line representing Venus' path, answering to the retrograde motion of the planet across the sun's disc of about four minutes of arc per hour.

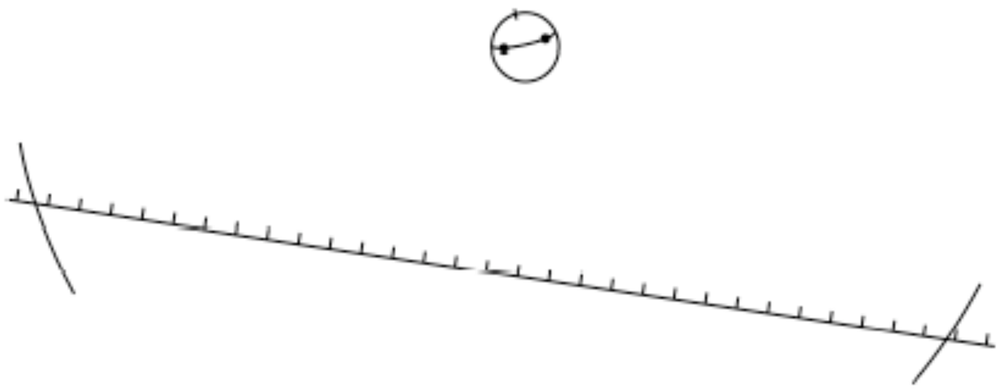

Figure 1. Halley's geometrical construction

For an observer, whose times of contact were to be found, an elliptical parallel of latitude was drawn within this circle

> … in the manner now used by Astronomers for constructing solar eclipses. [11]

Subsequently, a circle with a radius equal to the difference of the semidiameters of the sun and Venus was described round the point, representing the position of the observer on this parallel at the time of interior contact as seen from the earth's centre. This circle would then meet the visible path of Venus in a point denoting the time of interior contact as seen by the observer. The chief drawback of this graphical approach was that an accurate measurement of the apparent times of interior contact at the ingress and egress was hampered by the minuteness of the parallax of Venus from the sun compared to the sun's angular semidiameter, even when a large scale diagram was constructed. However, for his own predictions presented in the paper, whose accuracy could impossible be obtained graphically, Halley made use of

> … a good calculation, which, that I may not tire the reader, it is better to omit. [12]

Apparently, Halley had also divised an algebraic method to determine the effects of parallax upon the duration of the transit, but for the sake of the readers, who were thought not yet having themselves made acquainted with what Halley called the doctrine of parallaxes, an exposition of this method was not given. In omitting such explanatory excursions and avoiding intricate calculations, he clearly addressed his paper to those young people who had just little understanding of astronomy, but could well be fiery advocates of his proposal when the time of the transit had arrived. Therefore, he put his practical method in simple words and added a warm recommendation, with no intention to give it an exhaustive treatment from a mathematical point of view. Moreover, at the very end of his dissertation Halley indicated that he would further elaborate on his proposed method in due course:

> But how this parallax may be deduced from observations somewhere in the *East-Indies*, in the year 1761, both of the ingress and the egress of Venus, and compared with those made in its going off with us; namely, by applying the angles of a triangle given in specie to the circumference of three equal circles; shall be explained on some other occasion. [13]

However, Halley would never come back on the subject again in any publication. Yet the animated setting out of the foundations of his idea sufficed for giving an initial impetus to the further development of the method of durations by other astronomers.

Notes

[1] James Gregory. *Optica Promota*, 130. London: S. Thomson, 1663. English translation by the author.
[2] Eugene Fairfield MacPike (ed.). *Correspondence and Papers of Edmond Halley,* 40. Oxford: Clarendon Press, 1932. The bracketed words appear as symbols in the original text.
[3] Edmond Halley. "Mercurii transitus sub Solis disco, Octob. 28. anno 1677. cum tentamine pro Solis Parallaxi." *Catalogue des estoilles australes. Ou supplement du catalogue de Thycho*, 78. Paris: Jean Baptiste Coignard, Imprimeur du Roy, 1679. The original Latin text is provided as well by this contemporary French translation of Halley's *Catalogus Stellarum Australium, supplementum catalogi Thychonici*. London: Thomas James for R. Harford, 1679. English translation by the author.
[4] Ibid.: 80–2.
[5] Edmond Halley. "De Visibili Conjunctione Inferiorum Planetarum cum *Sole*, Dissertatio Astronomica." *Philosophical Transactions of the Royal Society* 17 (1691): 519. English translation taken from C. Hutton, G. Shaw and R. Pearson. *The Philosophical Transactions of the Royal Society of London, from their Commencement, in 1665, to the Year 1800; abridged, with Notes and Biographic Illustrations* 3, 454. London: C. and R. Baldwin, 1809.
[6] Ibid.: 522 (455–6).
[7] Ibid.: 522 (456). Last sentence translated by the author, as it is missing from the abridged *Transactions*.
[8] Edmond Halley. "Methodus singularis quâ *Solis* Parallaxis sive distantia à *Terra*, ope *Veneris* intra *Solem* conspiciendae, tuto determinare poterit." *Philosophical Transactions of the Royal Society* 29 (1716): 460. English translation taken from James Ferguson. *A plain method of determining the parallax of Venus, by her transit over the sun: and from thence, by analogy, the parallax and distance of the sun, and of all the rest of the planets*, 21. London, 1761.
[9] Ibid.: 460 (22). Ferguson says that the sun's parallax will '… be greater or less, *nearly* in the same proportion'. I omitted the word *nearly* because it is absent in the Latin text: 'Quod si major vel minor reperiatur ex observatione haec differentia, in eadem fert ratione major vel minor erit Solis parallaxis' (though the verbal form *fert* could perhaps imply the *nearly*).
[10] Ibid.: 457 (17–8).
[11] Ibid.: 462 (23).
[12] Ibid.: 463 (24). Bert Browne presents a calculation procedure that might well resemble Halley's method of calculation: L.W.B. Browne. "Halley's Method for Calculating the Earth-Sun Distance.*" Arch. Hist. Exact. Sci.* 59 (2005): 251–266
[13] Ibid.: 464 (26).